

# National Numeracy Review Report

May 2008

Commissioned by the Human Capital Working Group,  
Council of Australian Governments



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# Acronyms

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<b>AAMT</b>	Australian Association of Mathematics Teachers
<b>ACER</b>	Australian Council for Education Research
<b>ACDS</b>	Australian Council of Deans of Sciences
<b>AESOP</b>	An Exceptional Schooling Outcomes Project
<b>AMSI</b>	Australian Mathematical Sciences Institute
<b>AIM</b>	Achievement Improvement Monitor
<b>ARC</b>	Australian Research Council
<b>ASISTM</b>	Australian School Innovation in Science, Technology and Mathematics
<b>ATN</b>	Australian Technology Network of Universities
<b>CAI</b>	Computer Assisted Instruction
<b>CAS</b>	Computer Algebra Systems
<b>CBMS</b>	Conference Board of the Mathematical Sciences
<b>COAG</b>	Council of Australian Governments
<b>DEEWR</b>	Department of Education, Employment and Workplace Relations
<b>DEST</b>	Department of Education, Science and Training
<b>DET</b>	Department of Education and Training
<b>EMU</b>	Extending Mathematical Understanding
<b>ENRP</b>	Early Numeracy Research Project
<b>ESD</b>	English Spoken Dialect
<b>ESL</b>	English as a Second Language
<b>HCWG</b>	Human Capital Working Group
<b>ICT</b>	Information and Communication Technologies
<b>LBOTE</b>	Language background other than English
<b>MCEETYA</b>	Ministerial Council for Employment, Education, Training and Youth Affairs
<b>MERGA</b>	Mathematics Education Research Group of Australasia
<b>NCLB</b>	<i>No Child Left Behind</i>
<b>NDP</b>	<i>Numeracy Development Project</i>
<b>NESB</b>	Non-English speaking background
<b>NRA</b>	National Reform Agenda
<b>OECD</b>	Organisation for Economic Cooperation and Development
<b>PISA</b>	<i>Programme for International Student Assessment</i>
<b>SAT</b>	Standardized Aptitude Testing
<b>SES</b>	Socio-economic status
<b>SET</b>	Science, Engineering and Technology
<b>SiMERR</b>	National Centre for Science, Information and Communication Technology and Mathematical Education in Rural and Regional Australia
<b>TAFE</b>	Technical and Further Education
<b>TIMSS</b>	<i>Trends in International Mathematics and Science Study</i>
<b>UK</b>	United Kingdom
<b>US</b>	United States
<b>VCE</b>	Victorian Certificate of Education
<b>ZPD</b>	Zone of Proximal Development

# Preface

---

The Review provided an opportunity for a stocktake of research-based evidence about good practice in numeracy and the learning of mathematics. While there is a need to improve performance for some disadvantaged groups, overall we know that Australian students perform very well in international comparisons of performance. Our educators are held in high esteem and many overseas countries look to Australian educators for advice in the reform and renewal of their own systems.

This review occurred, not in a context of failure, but rather in the context of the need for on-going improvement and to ensure that we can continue to develop the numeracy and mathematical competencies needed for the human capital required to ensure our future prosperity as a successful modern society. International testing programmes have led to strong policy responses from countries whose performance is less than expected. We cannot assume that we can continue our competitive position without paying attention to emerging needs for maintaining and improving our teacher workforce.

The Review was carried out in the context of national agreements that Ministers have made about numeracy. For several years there have been assessments of numeracy with respect to national benchmarks at Years 3, 5 & 7 and from 2008 there is to be a national test of numeracy at Years 3, 5, 7 & 9 which will be reported at several proficiency levels. In addition Ministers have agreed to national Statements of Learning in Mathematics which are to be incorporated into all state curriculums by 2008. As indicated by the language used in this recent decision making there are questions to be resolved in relation to the distinction between numeracy and mathematics. Furthermore there will be a need to check that the sequential policy implementation process has not led to inconsistent outcomes in relation to the human capital requirements.

An important requirement for quality outcomes for education systems is that there is appropriate alignment between national policy agreements, curriculum and assessment practices and classroom pedagogy. As Porter (1994) points out, whether they are developed at the school-, system- or state-level, the success of education policies in bringing about change in practice depends in part on the consistency of the policies where 'consistency reflects the degree to which different education policies all call for the same education practice' (p 438). Not surprisingly, the effects of policies are greatest when they are mutually reinforcing.

Clarification of the numeracy/mathematics distinction is essential if we are to achieve national consistency in curriculum and outcomes and if the national assessment programme is to provide meaningful feedback about student progress. It is essential that actual assessment reflects what we really want and that it does not reinforce poor practice.

While literacy has received an enormous amount of attention and resources in recent years numeracy in some ways provides a bigger challenge for education systems. Kilpatrick et al (2001) made some important observations about the differences between the foundations of literacy and numeracy.

They point out that reading is based on a core set of representations which allows children to decode any English sentence even if the meaning is not fully understood at first. The capacity to develop greater understanding is enhanced by increasingly using these reading skills both within and outside the school environment.



In contrast to reading, mathematics has many types and levels of representation which build on one another as mathematical ideas become more abstract. While students can develop and use basic concepts outside of the school environment, a new and unfamiliar topic such as the division of fractions usually requires the assistance of a teacher or someone who can help the student access and understand the topic.

While school based instruction probably plays a bigger role in most children's development of mathematics than in reading, in recent years there has been greater energy and resource put into the development of literacy. If satisfactory outcomes are to be achieved for all more resources need to be directed towards improving mathematics teaching so that it supports the national goals of increased capacity building of numeracy skills.

For me it has been a great privilege to Chair the Review. My colleagues on the Review Panel have brought a scholarly and professional perspective to a task that has been carried out under an impossibly tight time-line. The difficulty of time pressure was not helped by the fact that all Panel Members have busy 'day jobs', live in different cities, and had to interact with a Secretariat based in Canberra. The practical difficulties in communication that this caused tested good-will on a number of occasions, especially when email communication proved less reliable than hoped for. Despite these difficulties the importance of the Review and the opportunity to express the significance of the numeracy agenda for national human capital capacity building has sustained us throughout the endeavour.

A handwritten signature in blue ink that reads "Gordon Stanley". The signature is written in a cursive, flowing style.

**Professor Gordon Stanley**

**Chair, Review Panel**

# Acknowledgements

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I would like to acknowledge the following people and organisations for their contribution to the National Numeracy Review:

## Expert Panel

**Professor Doug Clarke** – Professor, School of Education, Australian Catholic University

**Professor George Cooney** – Professor, Australian Centre for Educational Studies, Macquarie University

**Dr Thelma Perso** – Executive Director Curriculum, Education Queensland

**Professor Sue Willis** – Dean of Education, Monash University

## Reference Group

**Ms Jen Emery** – Department of Education and Children’s Services, South Australia

**Ms Debbie Efthymiades** – Department of Employment, Education and Training, Northern Territory

**Ms Trish Wilks** – Department of Education and Training, Australian Capital Territory

**Ms Glenys Reid** – Department of Education and Training, Western Australia

**Ms Dianne Peck** – Department of Education and Training, Victoria

**Ms Gillian Shadwick** – Department of Education and Training, New South Wales

**Ms Denise Neal** – Department of Education, Tasmania

**Mr John Boustead** – Department of Education, Training and the Arts, Queensland

## Expert Writers

**Dr Rachel Wilson** – Director, Educational Consulting Services

**Australian Council for Educational Research** – Dr Sue Thomson and Dr Michele Lonsdale

## National Forum Facilitator

**Professor Barry McGaw AO** – McGaw Group Pty Ltd

## Background Paper

**Professor Peter Sullivan and the Science, Mathematics and Technology team** – Monash University

## Secretariat Services

**Department of Education, Science and Training** – Ms Katy Balmaks, Ms Shona McQueen, Ms Toni Brown, Dr Lyndal Groom, Ms Nerida Rixon, Ms Belinda Veigli, Ms Tina Connor.

**Secretariat Australia Pty Ltd**

## Stakeholders

I would also like to thank all the stakeholders and individuals who provided submissions for the Review, and to all those who attended the National Forum.



# *Executive summary and recommendations*

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At its 14 July 2006 meeting, the Council of Australian Governments (COAG) reaffirmed its 10 February 2006 commitment to progress the National Reform Agenda (NRA), including the human capital agenda. This agenda is a long-term and integrated reform agenda across governments and portfolios, with the objective of increasing the nation's productivity and workforce participation.

COAG agreed that one of the initial priority areas would be literacy and numeracy – with the aim of improving student outcomes on literacy and numeracy. Literacy and numeracy skills are strongly correlated to success in school, students staying at school to Year 12 and to successful transition into further education and work. In July 2006 COAG tasked Senior Officials with completing specific reform proposals in the initial priority areas. Senior Officials have identified several proposals to enhance literacy and numeracy outcomes and further proposals are under development.

The Human Capital Working Group (HCWG) of COAG has commissioned this National Numeracy Review to synthesize information into a publicly accessible format on numeracy teaching. This is to include identification of the evidence available in relation to current and significant research including directions for teacher standards to improve the teaching of numeracy.

## **Improving numeracy outcomes for all**

Our review of national and international research and practice informs us that the mathematical knowledge, skill and understanding people need today, if they are to be truly numerate, involves considerably more than the acquisition of mathematical routines and algorithms, no matter how well they are learned. Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable.

This poses a substantial problem of trying to teach more mathematics in less time and given the considerable variation in the time allocated across schools and grade levels and the overall belief that time on task for mathematics has diminished over the years, action needs to be taken to ensure that there is an appropriate time allocation for mathematics. If Australia aspires to be one of the very high performing countries it has to decide what investment it is prepared to make and what it should prioritise.

Whether the goal is the learning of mathematics as such, or the development of numeracy more broadly, the rush to apparent proficiency at the expense of the sound conceptual development needed for sustained and ongoing mathematical proficiency must be rejected, as must the common propensity in Australian mathematics classrooms for assigning low level procedural tasks to students. The time, understanding and thoughtful action that deep mathematical learning requires must be acknowledged, and therefore both curriculum emphases and assessment regimes should be explicitly designed to discourage a reliance upon superficial and low level proficiency.

**To these ends, we recommend:**

1. That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. Both pre- and in-service teacher education should thus recognise and prepare all teachers as teachers of numeracy, acknowledging that this may in some cases be 'subject specific numeracy'. (refer to p.7)
2. That all jurisdictions should work towards a minimum of 5 hours per week of mathematics for students in all the primary Years K to 6/7 and a minimum of 4 hours per week in all the lower secondary Years 7/8 to 10. This time should include cross curricular learning. (refer to p.18)
3. That from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking. (refer to p.31)
4. That a balanced view be taken of the relative contributions to effective student learning of systemic assessment programmes and high quality classroom assessment in the allocation of resources to develop and support each. (refer to p.42)

While overall levels of numeracy/mathematics achievement in Australia are quite good by international standards, there is an unacceptable proportion of Australian students (particularly but certainly not only amongst Indigenous students) who are not achieving acceptable levels of proficiency. Many students also lack confidence in the subject, do not enjoy or see personal relevance in it and are unlikely to continue its study voluntarily. This clearly is a risk to Australia achieving its human capital goals, but the personal and social consequences for individuals and their families and communities can be unfortunate in ways that go beyond the purely economic.

In Australia, targeted interventions tend to be directed at students identified as at risk of not meeting the National Benchmarks. These, at least at Years 3 and 5, assess minimum standards rather than desirable levels of proficiency, the implication being that minimum standards are good enough, at least for some students. All students and their families, however, have a right to expect high quality, not minimum, numeracy outcomes from their schooling.

**To these ends, we recommend:**

5. That the necessary resources be directed to support teachers to use diagnostic tools including interviews to understand and monitor their individual students' developing strategies and particular learning needs. These diagnostic tools should not be restricted to school-entry assessments. (refer to p.42)
6. To raise the overall level of achievement, increased resources (including specialist teachers working 'shoulder to shoulder' with teachers) should be directed to support teachers in regular classrooms to provide intervention for a higher proportion of students during all the compulsory years of schooling, and that:
  - the focus of intervention for students at risk be on enabling every student to develop the in-depth conceptual knowledge needed to become a proficient and sustained learner and user of mathematics
  - these resources be particularly focused on the early years of schooling. (refer to p.62)
7. That systemic assessment programmes be extended to include sampling of students to provide more in depth information about common conceptions and misconceptions, and areas of difficulty for students, with the purpose of providing (a) a research base to inform ongoing curriculum development and pedagogy and (b) improved diagnostic tasks for individual teacher use with students in their classrooms. (refer to p.42)
8. That the language and literacies of mathematics be explicitly taught by all teachers of mathematics in recognition that language can provide a formidable barrier to both the understanding of mathematics concepts and to providing students access to assessment items aimed at eliciting mathematical understandings. (refer to p.34)
9. That the use of ability grouping across classes in primary and junior secondary schooling be discouraged given the evidence that it contributes to negative learning and attitudinal outcomes for less well achieving students and yields little positive benefit for others, thus risking our human capital goals. (refer to p.49)

## Teacher standards and professional learning

Teachers are the key to effective pedagogies that serve the needs of all students in all circumstances. Good teaching cannot be made routine or substituted by texts and teaching materials. It requires deep and connected knowledge on the part of teachers, the exercise of complex and high level judgments both cognitive and interpersonal, and a well-informed and varied repertoire of strategies appropriate for the learning of mathematics.

In the last ten years in Australia, there has been a range of innovative, research-based professional development programmes for teachers and support staff in the early years of schooling, some extending into the middle years. Whether directed at improving the learning of children generally, or focussed on particular groups identified as potentially 'at risk', these exemplary programmes have certain features in common.

It is clear that a collaborative environment plays an important role in professional learning, whether in teams in primary schools or departments/teams in secondary schools. Experienced teachers have a key role to play in mentoring less experienced teachers and should be supported in the school environment.

**To these ends, we recommend:**

- 10.** That the Australian Association of Mathematics Teachers (AMMT) Standards for Excellence in Teaching Mathematics in Australian Schools be used as a framework for professionalism in the teaching of mathematics and inform the development of the forthcoming national numeracy teaching standards. (refer to p.73)
- 11.** That the research-based professional development programmes identified in this report as exemplary in supporting early and primary years' teachers to enhance numeracy outcomes be extended in their reach and impact; further that these programmes or others developed on similar principles be extended to include teachers of students up to Year 10. Exemplary professional development programmes are based on:
  - enhancing pedagogical content knowledge (that is, knowledge about teaching specific mathematical content)
  - providing teachers and support staff with approaches for accessing the thinking of individual students
  - the premise of high expectations of all students and provide conceptually rich strategies for addressing the needs of those not achieving well
  - a strong theory-practice link including partnerships between schools, systems and universities
  - providing sustained opportunities for teacher learning and reflection and collegial and/or specialist support. (refer to p.75)
- 12.** That pedagogical content knowledge (that is, knowledge about teaching specific mathematical content) be a prime focus of both pre-service and in-service programmes for teachers of mathematics across all the years of schooling. (refer to p.75)
- 13.** That all teachers of mathematics and numeracy be equipped to identify and understand how personal circumstances, cultural practices and the particular mathematical needs of individual students may impact upon their learning of mathematics, and to intervene as necessary, drawing on a repertoire of effective pedagogies to ensure that these learning needs are met. (refer to p.64)
- 14.** That, in recognition of the likely continued reliance in the medium term on teachers teaching secondary mathematics 'out of field', systems develop strategies to support such teachers to improve the depth and extent of their mathematical and pedagogical content knowledge. (refer to p.75)
- 15.** That structured programmes be implemented to support teachers to develop the knowledge and skills necessary to exercise effective leadership roles in numeracy and mathematics within schools. (refer to p.77)

# Chapter 1: Numeracy and mathematics in Australia

---

## 1.1 Introduction

Mathematics is not generally perceived as a popular subject among young people, despite movies and television shows in recent years focussing on the use of mathematics or having mathematicians as central figures. Mathematics is also not recognised as an easy subject to learn or to teach. There is, however, strong and continuing interest in increasing levels of numeracy within the community where, in a very general sense, numeracy is associated with the mathematical problem-solving skills, understandings, and applications needed in both today's and future society.

The National Numeracy Review takes place against a background of increasing globalisation, rapidly changing technology, an increasing sense of insecurity on almost every front, and media-fuelled debates about how well our education system is preparing Australian students for these current and future challenges. Rowe [47], for example, argues that the global economic, technological and social changes under way, requiring responses from an increasingly skilled workforce, make high quality educational provision an imperative – especially high quality teaching.

That schools, teachers and teacher education are charged with being all things to and for all young people is hardly new. Over seventy years ago, a Director of Education in Western Australia wrote in the forward to the new 'Green Syllabus', 'It may sound trite and platitudinous, but it is as true today as ever that as is the teacher, so is the school,' continuing:

No matter what our education policy may be, whether we go to the right with the defenders of the old, to the left with the most advanced of the progressives or down the middle of the road with the majority, we depend upon the teachers to carry our principles through. ... They alone can, by the successful training of the youth committed to their care, build a newer, saner, wiser social order (Klein 1936, p.6).

Expecting of teachers and schools that they 'alone' be responsible for building 'a newer, saner, wiser social order' is a rather tall order and possibly a reflection of the educational and political thinking of the time. Nevertheless, the power that schools have to change lives cannot be underestimated nor taken lightly. Indeed, the constant press that schools, teachers and teacher education get could be interpreted as recognition of the considerable significance of education in 'making the difference' both for the nation as a whole and for the individuals and social groups that form it.

The available evidence points to better educational and labour market outcomes generally for those with good levels of numeracy. Such outcomes include the following:

- Completion of Year 12: Literacy and numeracy achievement are the strongest predictors of Year 12 completion.
- Successful transitions from school: The best transitions are achieved by those with high levels of numeracy (around 95% have good transitions), ahead of those with high levels of literacy (92%). The higher the level of literacy and numeracy, the higher the probability of labour force participation and the lower the probability of unemployment.



- Participation in post school education and training: Those with strong literacy and numeracy skills are more likely to go to university or other education and training after leaving school. This enhances the chances of further skill development throughout life.
- Labour market outcomes: There is a positive relationship between numeracy skills and wages/earnings (see e.g. Fullarton et al., 2003; Lamb & McKenzie, 2001; McMillan & Marks, 2003).

In this broader context of expectations upon the education system, and of the particular significance of numeracy and literacy, questions arise as to how well the Education system serves the interests of Australian students in relation to numeracy. Commonly expressed concerns about Australian student levels of numeracy include that:

- Australian students are not learning the ‘basics’ and are thus not being equipped adequately for either further study or future employment
- Australian students do not perform well relative to other countries
- Australia has a long ‘tail’ of underachievement in international tests
- there are pockets of low achievement reflecting particular socio-economic, geographical, cultural and racial/ethnic factors
- student numbers in mathematics at both senior secondary and tertiary levels are declining with serious workforce implications for the future.

These concerns will be addressed both directly and indirectly in the remainder of this chapter.

## 1.2 Numeracy and the school curriculum

In the 2007 Parents’ Attitudes to Schooling Survey, a telephone based random sample of over 2000 parents of school age children, the great majority of parents (91.0%) believed that there was certain content that all children should learn at school during the compulsory years of their education (DEST, 2007). They particularly highlighted ‘mathematics’ (81.1%), indicating that it was ‘very important’. Similarly, the 2003 Parents’ and Community Members’ Attitudes to Schooling Survey found that parents and community respondents regarded it as ‘very important’ that schools assist children in developing numeracy (parents, 85.4%, community, 77.9%) (DEST, 2003). What is perhaps surprising about this is that between 15% and 19% of parents and 22% of the broader community did not think so.

What this survey is unable to tell us is what parents and the broader community thought the terms ‘numeracy’ and ‘mathematics’ meant and what skills, understandings and attributes they expected children to develop as a result of their school education. Of particular concern is that in the 2007 survey, fewer than half of the surveyed parents believed that students were leaving school with adequate skills in numeracy (39.8%). We do not know what numeracy skills these parents believed to be missing or the source of, or evidence, for their concern. Was it that what their children were learning was no longer comfortably familiar or that it was still all too familiar? Was it children they actually knew who they believed lacked numeracy skills, or teenagers serving in the nearby fast food outlet, or those they had heard about in the media? We do not know. What we do know, however, is that the majority of those who responded to the survey lacked confidence in what schools were delivering in relation to numeracy.

As indicated later in this chapter, the international test evidence suggests that on the whole Australian students perform quite well in comparison with other countries. This may, of course, tell us little more than that problems of innumeracy are a worldwide phenomenon. Indeed, almost twenty years ago,

Paulos (1988) alerted the international community to the perils of innumeracy pointing to a potentially serious problem, as societies become more and more dependent on mathematical models for problem solving. Mathematics curricula and pedagogy are often described as not responding adequately to the need to provide the pertinent mathematical knowledge required of the citizens of today and tomorrow, a knowledge that will result in an ability to choose and use the mathematics learned to meet personal and social goals.

## Numeracy and mathematics

Numeracy is at times thought of as a subset of school mathematics, the 'basic mathematics' needed for every day or perhaps the basic building blocks of school mathematics, the foundations, and at other times as somewhat more than mathematics, involving a grasp of the interplay between mathematics and the social contexts within which it is used. Clearly there are ambiguities, with 'mathematics' and 'numeracy' being used almost interchangeably at times and at other times regarded as quite distinct.

### The meaning of numeracy

The Ministerial Council for Education, Employment, Training and Youth Affairs (MCEETYA) in its 1997 *National Report on Schooling in Australia* stated:

'Numeracy is the effective use of mathematics to meet the general demands of life at school and at home, in paid work, and for participation in community and civic life' (MCEETYA 1997, p.130).

This follows the work of Willis who, in 1992, defined being numerate as 'being able to use mathematics – at work, at home, and for participation in community or civic life' (p.5) qualifying the definition and its implications as follows:

... this rather modest proposition is unlikely to meet with much disagreement... Nevertheless, to take it seriously would have dramatic consequences for the practice of mathematics education in a great many educational settings. It would suggest that numeracy is not about the acquisition of even a large number of decontextualised mathematical facts and procedures. .... It would suggest that numeracy is about practical knowledge where practical should not be confused with low level, 'hands on' or procedural knowledge. I am using the term 'practical knowledge' here to refer to knowledge which has its origins and/or importance in the physical or social world rather than in the conceptual field of mathematics itself (pp.5-6).

From this perspective, numeracy is regarded as 'the capacity to bridge the gap between 'mathematics' and 'the real world', to use in-school mathematics out-of-school' and people are considered more or less numerate based on 'how well they choose and use the mathematical skills they have in the service of things other than mathematics' (Willis 1998, p.37).

In the past decade, there has been some convergence of views at least in Australian curriculum policy and the mathematics education research literature about what numeracy entails. The AAMT describes numeracy as involving:

... the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic); mathematical thinking and strategies; general thinking skills; [and] grounded appreciation of context (AAMT 1997, p.15).

From the United Kingdom (UK):

Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value (Evans 2000, p.236).

From the Netherlands:

Numeracy encompasses the knowledge and skills required to effectively manage mathematical demands in personal, societal and work situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology (Van Groenestijn 2002, p.37).

The commonality in these definitions is clear. The implications for the mathematics curriculum or for the curriculum more broadly are less obvious. AAMT describes the relationship between school mathematics and numeracy thus:

Numeracy is not a synonym for mathematics, but the two are clearly interrelated. All numeracy is underpinned by some mathematics; hence school mathematics has an important role in the development of young people's numeracy. The implemented mathematics curriculum (i.e. what happens in schools) has a responsibility for introducing and developing mathematics, which is able to underpin numeracy. However this 'underpinning of numeracy' is not all that school mathematics is about. Learning mathematics in school is also about learning in the discipline – its structure, beauty and importance in our cultures. Further, while knowledge of mathematics is necessary for numeracy, having that knowledge is not in itself sufficient to ensure that learners become numerate (1997, pp.11-12).

### **Mathematical literacy**

Over recent years, the expression 'mathematical literacy' has become more widely used internationally as the term 'numeracy' does not easily translate into some languages. For example, the highly influential *Programme for International Student Assessment (PISA)* which will be described in the next chapter, defines *mathematical literacy* as:

... an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgements, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (OECD 2004, p.15).

This definition clearly echoes those of numeracy described above.

Jablonka (2003) provides a powerful overview of mathematical literacy, arguably the most comprehensive available to date. She identified five distinct trends in the mathematical literacy research literature each involving increasingly sophisticated mathematical demands, they are: mathematical literacy for developing human capital, cultural identity, social change, environmental awareness and evaluating mathematics. In the context of this review, the first of these has particular relevance. Mathematical literacy for developing human capital has at its heart the economic argument for numeracy education, that is, the needs of society are changing and in order for the country to maintain its lifestyle and economic well-being we need better and more mathematically educated adults and school leavers. It recognises that we are not just talking about mathematicians, that we need to ensure that all school leavers, with whatever specialised futures in mind, are nonetheless also powerfully numerate in order that they will be equipped to contribute effectively to the development of our society.

## Numeracy and ‘the basics’

Notwithstanding the above interpretations, for many members of the broader community and indeed for many teachers and policy makers, the term ‘numeracy’ is used more or less synonymously with mathematics, or even with the ‘basics’ of mathematics, particularly in the context of public commentary about ‘numeracy standards’. The important questions then become ‘which basics?’ and ‘standards of what and for whom?’ Again, this is no obvious matter.

On the one hand, there is the notion that the ‘basics’ are what we might regard as ‘functional numeracy’, that is, everyday fluency with arithmetic and measurement and perhaps the capacity to find one’s way around. The *National Statement on Mathematics for Australian Schools*, captured part of this as follows:

All school leavers should feel confident in their capacity to deal with the computational situations which they meet daily and number work should reflect the balance of number techniques in regular adult use (Australian Education Council and Curriculum Corporation 1991, p.108).

Many people still believe the standard written procedures they learned at school are what is necessary for computational proficiency, often in the face of their own personal experience. Indeed, the evidence is that adults rarely use formal written procedures but rather use a combination of mental arithmetic, calculators and informal ‘back of the envelope’ jottings. For example, a decade ago, 200 adults of all ages and a wide range of educational and occupational backgrounds were surveyed and asked to record the calculations they performed over a typical 24-hour period (Northcote & McIntosh 1998). Their records showed the following pattern of use of these skills (the total is greater than 100% because sometimes they used a combination of two approaches for a single calculation):

Mental	84.6%
Written	11.1%
Technology	19.6% (or physical objects)
Calculations requiring estimate only	60%
Calculations requiring exact answer	40%

The *National Statement* took the view that young people need to develop sensible methods for calculating but that many of these sensible methods may be idiosyncratic (both to the individual and the particular task) and the majority are likely to be mental methods.

On the other hand, there is the notion that the ‘basics’ are the fundamentals of mathematics. As Lyn Arthur Steen, then President of the Mathematical Association of America, noted almost two decades ago:

The key issue for mathematics education is not whether to teach fundamentals but which fundamentals to teach and how to teach them. Changes in the practice of mathematics do change the balance of priorities among the many topics that are important for numeracy. Changes in society, in technology, in schools – among others – will have great impact on what will be possible in school mathematics in the next century. All of these changes will affect the fundamentals of school mathematics (Steen 1990, p.2).

It also raises the question of whether the ‘topics’ of mathematics, no matter how they are defined, or mathematical routines and algorithms, no matter how well they are learned, of themselves deliver numeracy. Twenty-five years ago, the Cockcroft Report (a three-year government inquiry into the teaching of mathematics in schools) made the now famous claim that for the great majority of students mathematics learned in isolation, remains in isolation (Cockcroft, 1982).

As suggested, many assume that if you've got the mathematical skills then you are numerate – more or less regardless of whether you use the skills or not. There is a substantial body of research, however, indicating that knowledge alone is hardly sufficient for problem solving or further learning.

Good problem solvers differ from poorer problem solvers not so much in the particular skills they possess as in their tendency to use them ... [and] acquiring skills and strategies, no matter how good one [becomes] at them, would not make one into a competent [user of them] (Resnick & Klopfer, 1989, p.6).

We should not make the mistake of confusing the acquisition of mathematical concepts and procedures with the 'competence and disposition to use mathematics' in context, that is, with numeracy.

As a professor of mathematics education stated during the consultations for this review, Australia has made the decision to go the 'numeracy route'. This encourages the teaching of mathematics in a manner that emphasises its application in many facets of everyday life. It also encourages the development of the 'using and choosing' skills necessary for the effective use of mathematics. Little evidence was found in the submissions to support a reversal of this approach, indeed there was considerable enthusiasm for continuing it.

## Numeracy across the curriculum

As a number of submissions to the review pointed out, numeracy, like literacy, is increasingly regarded as an 'across the curriculum' issue [e.g. Catholic Education Brisbane, 5; New South Wales Board of Studies, 34; Department of Education and Training (DET) Western Australia, 40]. There are two aspects to this, each important.

Firstly, for students to develop numerate behaviour they have to have opportunities to practise it. Students need to learn to ask themselves whether mathematics will help deal with a situation, to recognise when and how it might be used and to make judgements that are adapted to fit the context. Even when the notion of 'school mathematics' is broadened to include both mathematical knowledge and the more strategic applying processes characteristic of mathematics, it is unlikely to be able to capture fully all that is numeracy simply because the mathematics is *in* mathematics. Schools cannot offer students real out-of-school settings but they can provide access to a range of different contexts across the whole school curriculum. While the major responsibility for the enhancement of numeracy resides within school mathematics, numeracy outcomes for students will be enhanced by an across the curriculum focus premised on the principle that numeracy education is everybody's business.

Secondly, mathematics can make a difference to whether and how well students learn across the curriculum. Teachers of subjects other than mathematics need to understand the mathematical demands of the work they ask of students and the potential difficulties students might experience and they need to have strategies for addressing them. These demands could vary from a Year 1 child being able to understand what is going on in a story that refers to 'twice as many pigs on a pirate ship' being 30, to a Year 7 student trying to find the centre of a wooden circle in order to make a wheel, and to a Year 12 history student grappling with the difference between a 'state' and a 'rate' in order to understand why, during a plague, as 'mortality rose, fertility rose'.

In 2004, Hogan argued in the Department of Education, Science and Training (DEST) report, *Teachers Enhancing Numeracy*, that 'Good numeracy skills are important for learning across all curriculum areas and are essential for life after schools' (p.viii) and 'Knowledge of mathematics and knowledge of its application

in a range of contexts seems to provide students with confidence. These students are more prepared to have a go, make mistakes and try again' (p viii). Earlier, in 1997, the AAMT in its report '*Numeracy = everyone's business*', published in support of the National Plan, adopted the position that 'numeracy is cross-curricular and is a responsibility for all educators' (p 39). It argued that priority should be given to professional development programmes that (amongst other things) 'develop shared understandings of the numeracy demands across all learning areas, and of the responsibilities of all teachers to contribute to the development of students' numeracy' (p 41). A number of submissions to the review pointed to the work that has been undertaken in the decade since then in jurisdictions across Australia. These include, for example, mapping key aspects of numeracy across key learning areas [Catholic Education, Brisbane, 5], and systematically building aspects of numeracy into all K to 10 syllabuses including performance descriptors and assessment activities [New South Wales Board of Studies, 34] and research and development projects in schools relation to numeracy across the curriculum [DET Western Australia, 40].

### **Recommendation 1:**

That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. Both pre- and in-service teacher education should thus recognise and prepare all teachers as teachers of numeracy, acknowledging that this may in some cases be 'subject specific numeracy'.

## **Numeracy and the workplace**

Over the past two decades there have been many studies of out-of-school numeracy practices of adults (FitzSimons, 2002). Some have been functional in orientation, looking for evidence (or not) of the use of recognised school mathematics topics in the workplace and society. Others have adopted a situated cognition approach, the best known of which is Lave (1988) who observed various groups of people at work. She showed that the mathematical knowledge and skills utilised by shoppers and weight watchers, for example, bore little resemblance to the formal processes taught in school.

In more recent years there have been several large-scale studies on mathematics or numeracy in the workplace in the UK (e.g. Bakker, Hoyles, Kent, & Noss, 2006; Hoyles et al. 2002; Kent & Noss 2002; Wake & Williams 2001). In Australia, researchers have applied similar theoretical frameworks to conduct research on key competencies in the workplace (see, e.g. Kanes, 2002) and workplace numeracy (FitzSimons 2005a; FitzSimons & Wedege 2007). Collectively, these projects have important implications for numeracy needs of future Australian citizens suggesting that adult and workforce needs can be contrasted with the perceived intent of school curricula:

- In school the object of activity is for students to learn mathematics in a supportive environment, whereas in the workplace the object is to achieve a productive outcome under constraints of time, money, safety, legislative requirements, etc., and mathematics is but one tool or mediating artefact in this process.
- The mathematics used in the workplace is often invisible or viewed as relatively low-level when compared to lists of school mathematics topics, but it actually requires substantial depth of understanding with mistakes to be avoided at all costs.
- In the workplace, knowledge of context and content is of the essence. Judgements are made, often instantaneously, in light of all available quantitative and qualitative information, including historical records and sensory data on physical conditions as well as dynamic technology-generated data.



- Among the hybrid of generic competencies required in practice, communication plays a vital role, especially in times of breakdown in equipment or understanding, and it is at these times, the visibility of the mathematics can come clearly into focus.
- Knowledge and skills are not simply 'applied' but transformed with (locally) new knowledge created by adults as citizens and/or workers in response to unpredictable and ever-evolving problems. The transfer of school mathematical knowledge cannot be assumed.

There is a direct connection with these perspectives and human capacity building.

If numeracy is about using mathematics effectively to meet the general demands of life at school, at home, in paid work and for participation in community and civic life, then it is clearly the role of the school curriculum – both documented/planned and implemented/enacted – to enable young people to learn to use mathematics to meet these demands and 'to bridge the gap between mathematics and the real world' (Willis, 1998, p.37).

Of course this is not only the responsibility of schools, or only of significance to those making 'everyday' use of mathematics, since the 'real world' also includes a range of occupations in which sophisticated mathematical skills are required. At least one submission to this review, that of the Australian Technology Network of Universities, suggested that the university mathematics curriculum could be subjected to much the same criticisms as those above made of the school curriculum. They referred to a study of stakeholders and students in engineering that reached the following conclusion:

While engineering graduates will always need a proficient level of maths, ... Universities ... were still concentrating on many of the 'old' maths and science skills which were no longer required, or required to a much lesser degree. ... There was uniform agreement from all graduates in all focus groups that much of the mathematics included in their courses was never required in the workplace and that the time could have been better spent on other areas. Telecommunication engineers and aerospace engineers had the least positive comments [and] reported that it was their belief that their courses did not reflect what the workplace required and this was an issue for them in the workplace [Australian Technology Network of Universities (ATN), 6, p.2].

As the 2007 Parents' Attitudes to Schooling Survey shows, (almost) everyone knows that mathematics is important (DEST, 2007). Whether preparing for the broadly numerate workforce or the mathematically oriented professions, however, it appears that at all levels of education up to and including university, many students (and their parents) are not persuaded that the mathematics education they are receiving will serve them well in their future workplaces. In this context, it is not surprising if students who believe themselves to have other alternatives take those alternatives.

This raises a number of questions for school mathematics curricula and the curriculum more broadly. Firstly, do the mathematical needs of an adult in the workforce differ from that envisaged in the design of school curriculum or are curricula appropriate but the ways in which they are taught problematic? Secondly, is it possible for schools ever to mimic the complexity involved in the application of mathematical knowledge to real tasks in real workplaces? And thirdly, to 'which workplaces' are we referring? Scientists and mathematicians, for example, are also in the workforce and presumably their workplace requirements also need to be met. The question is not simply whether the school mathematics curriculum, as designed and/or as taught and learned, prepares people for the workplace, but rather which workplaces (and other places) it does and does not serve well and how we address the very different needs of adult life.

## Dimensions of a school curriculum for numeracy

A recurring theme in both the descriptors from numeracy projects and research studies identified in the literature review and the submissions was that a curriculum directed at producing a productively numerate population needs to provide experiences along a number of distinct dimensions of learning and using mathematics. Although the details vary, these dimensions could be thought of broadly in the following way:

- 1. Mathematical:** This is about learning the mathematical content which provide the models to be understood, analysed and applied, that is, the concepts, procedures and skills which comprise what we think of as 'school' mathematics, e.g. being able to calculate  $1.23 + 3.4$  mentally and also knowing without doing the calculation that  $1.23 + 3.4 = .157$  must be wrong because the answer must be more than 4; also knowing that to subtract 20% is the same as finding 80% or multiplying by 0.8.
- 2. Strategic:** This is about developing a repertoire of strategic mathematical processes, appreciations and dispositions needed to choose and use mathematics to solve familiar and unfamiliar problems. The attention shifts from the concepts and skills themselves to the processes needed for moving between the real and practical world and mathematics, e.g. confronted with the question of whether it is better to take the 20% discount from an item and then add the 10% GST or add the GST first and then discount, the person tries a few simple examples, and looks for commonalities, before trying to generalise. (Of course, the mathematical may come into play, here, when the person realises that adding 10% and subtracting 20% is the same as finding 90% of 120% of the price which is  $0.9 \times 1.2$  of the price and since multiplication is commutative, the order is not relevant to the customer.)
- 3. Contextual:** This is about experiencing opportunities to develop and apply mathematical knowledge in a range of situations, both familiar and unfamiliar, in order to develop an understanding of the way in which contextual features can determine the appropriateness and usefulness of particular mathematical approaches, e.g. confronted with the question of whether it is better to do the 20% discount or the 10% GST first, the person realises that it depends from who's perspective you are asking, it makes no difference to the customer but it does to the shop keeper and the tax collector. It also comes into play when the person realises that, in this case, it is best to stick with the law.

The first dimension is clearly seen by many writers, researchers and practitioners as the most significant. A 'numeracy' curriculum would be meaningless without a strong mathematical dimension and in all the writings about numeracy; the mathematical 'backbone' of the subject has been present, or at least assumed.

The studies of workplace numeracy demands quoted earlier; do suggest that from adult and work place perspectives, the school curriculum should provide:

- a deep understanding of the real number system and its links with the metric system of measurement so that this knowledge is embodied rather than a series of disconnected and often incorrectly recalled facts. Similar understanding for statistical, geometrical and algebraic thinking is also recommended.
- experience grounded in practical situations of making contextualized judgements about levels of accuracy, reasonableness of answers, and when to approximate
- experience in the use of non-standard artefacts – e.g. charts, tables, electronic databases, internet support, as well as working interactively and creatively with spreadsheets



- experience working in inter-disciplinary (or inter domain) project teams to incorporate the range of generic competencies (e.g. problem solving, working in teams, communication, technology skills) working within realistic constraints
- with respect to lifelong learning, an ongoing willingness to question existing practices, to learn new skills, and to have the confidence to make mathematical and other evidence-based recommendations concerning aspects of existing practice.

Notwithstanding such work, one of the striking features of the numeracy literature is the lack of creative research about the necessary, or possible, mathematical content most likely to support rich numeracy practices. The content of school mathematics curricula continues largely to ignore such work and to reflect opinion-based 'settlements' about what matters for whom.

One significant offering in the literature is a book edited by Steen (1990), *On the Shoulders of Giants: New Approaches to Numeracy*. It attempts to define the new 'foundations' and contains five chapters by five different mathematicians, who 'were asked to explore ideas with deep roots in the mathematical sciences without concern for limitations of present schools or curricula' (p.iii). The resultant chapters are called Dimension, Quantity, Uncertainty, Shape, and Change. This book emanated from the Mathematical Science Education Board of the United States (US), and though its contribution may seem to be highly mathematical and content-specific, it nevertheless offers a powerful idea of the curricular need for defining not just procedures, but also concepts and fields.

It is worth noting that PISA adopted the content descriptors from Steen, but nevertheless uses generic terms in its descriptors of levels of achievement for the content dimension and does not specify algebra or geometry or any other specific concepts or fields:

Students at level 6 can ...

- conceptualise, generalise, and utilise information based on their investigations and modelling of complex problem situations
- link different information sources and representations and flexibly translate among them
- utilise advanced mathematical thinking and reasoning
- apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations
- formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.

Students at level 3 can ...

- execute clearly described procedures, including those that require sequential decisions
- select and apply simple problem solving strategies
- interpret and use representations based on different information sources and reason directly from them
- develop short communications reporting their interpretations, results and reasoning.

Students at level 2 can ...

- interpret and recognise situations in contexts that require no more than direct inference
- extract relevant information from a single source and make use of a single representational mode

- employ basic algorithms, formulae, procedures, or conventions
- use direct reasoning and make literal interpretations of the results.

This does not imply that PISA wishes content to be ignored or that the traditional mathematics curriculum should be discarded. It is, however, a significant challenge to specify and justify in detail the most relevant mathematical models and concepts and gain ‘universal’ agreement.

In summary, the PISA definition of mathematical literacy is informed by a view of the role of numeracy in building an individual’s personal, social, and economic well-being, and of how adults use their mathematical knowledge, skills and attitudes to make decisions and judgments in everyday life. It suggests that the mathematical knowledge, skill and understanding people need today, if they are to be truly numerate, involves considerably more than the acquisition of mathematical routines and algorithms, no matter how well they are learned. Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context.

### 1.3 Achievement in mathematics

In Australia, as part of the NRA, of which the human capital agenda is a component, a set of Key Performance Measures have been identified to indicate the extent to which the National Goals of Schooling are being achieved (MCEETYA, 2003). In relation to numeracy, the Key Performance Measures now include light sample international tests, the *PISA and Trends in International Mathematics and Science Study (TIMSS)*, and full-cohort national tests of numeracy to be reported against a set of standards which include the National Numeracy Benchmarks and National Statements of Learning (MCEETYA, 2005).

#### International comparative assessments

McGaw (2007a) argued that international comparative assessments encourage systems to strive for quality and equity, broaden the search for best practice, and encourage a focus on the longer-term intentions of curriculum. McGaw also identified risks in such assessments including the narrowing of the curriculum because assessments are limited in scope and results can oversimplify the complex differences between systems.

The PISA is an Organization for Economic Cooperation and Development (OECD) initiative managed by a consortium led by Australian Council for Education Research (ACER) and implemented on a three-year cycle. TIMSS is run by the International Association for the Evaluation of Educational Achievement and conducted on a four-year cycle. Participation in these studies enables benchmarking of the performance of Australian students against students in other countries, to monitor student performance over time and to examine the relationships between various background and contextual factors and student performance. The most recent results available are from PISA 2003 and TIMSS 2002/03.

PISA focuses on the abilities of students nearing the end of their compulsory schooling, to apply their understandings and skills to real-life problems and situations, while TIMSS looks at how well students have mastered the factual and procedural knowledge taught in school mathematics curricula. Also PISA assesses an age-based sample of 15-year-olds while TIMSS uses a grade-based sample at Year 4 and

Year 8. There are also different countries and different numbers of countries participating in the two assessments. PISA is mostly focused on developed OECD countries, but with an increasing number of non-OECD countries participating, while there is a broader range of countries in TIMSS.

The focus of the PISA assessment programme is on how well young people have been prepared to meet challenges, how well they can adapt their learning to the needs of their lives, and aspects of school organization, including factors contributing to disadvantage. Consistent with the first two foci, the PISA items are not restricted to narrow aspects of numeracy learning. For example, in one question from the problem solving assessment, students were presented with a photo on which there were three spaced footprints in sand. The text was:

The picture shows the footprints of a man walking. ... For men the formula  $= 140$  gives an approximate relationship between  $n$  and  $P$ , where  $n$  is the number of steps per minutes and  $P$  is the pace length in metres.

The question was posed as follows:

Bernard knows his pace length is 0.80 metres. ... Calculate Bernard's walking speed in metres per minute and kilometres per hour.

A rubric was used to score responses. For 15-year-old students this question represents a significant challenge, and it is clear that the assessment went beyond routine aspects of numeracy.

In 2003, Australia was one of 41 countries that participated in PISA. Australia can be well pleased by being significantly ahead of the OECD average and many individual OECD countries in its average level of performances in PISA. Results from the project carried out in 2003 place Australia equal 5th with eight others behind Hong Kong, Finland, Korea and the Netherlands (Thomson, Cresswell, & de Bortoli, 2004).

The OECD has suggested that countries that performed relatively better in problem solving than in mathematical literacy (as was the case for Australia) had students with the potential to achieve better results in mathematics than that reflected in their current performance, since their level of generic problem-solving skills is relatively higher. They further suggested that in countries that perform relatively higher in mathematical literacy than in problem solving, mathematical instruction may be particularly effective (OECD, 2004, p.55). This analysis, however, seems to suggest that what is assessed under 'problem solving' in mathematics is not a result of effective instruction while mathematical literacy is. It is equally arguable that each is a result of mathematics instruction, that each produces and is necessary for mathematical performance, and that greater time or attention to mathematical literacy might be at the expense of time and attention to mathematical problem solving.

McGaw (2007a) noted that, with respect to the mathematical literacy component, Australia was one of 11 countries with similar results, and only four countries outperformed this group. In problem solving, Australia was one of nine countries with similar results, and there were also only four countries that performed better. In other words, while not the top, the overall mathematical literacy and mathematical problem solving performance of Australian students as measured in this study is good. Of course, our goal is for Australia to achieve even better results in each.

The other major study, TIMSS, seeks to provide important information for policy development, and to allow areas of progress or decline in achievement to be identified and monitored, and to address concerns for equity. The following is an example of an item for the Year 8 students in 1994/5:

A rubber ball rebounds to half of the height it drops. If the ball is dropped from a rooftop 18m above the ground, what is the total distance travelled by the time it hits the ground the third time? (Item L-11, p.37, TIMSS 1994/5)

The TIMSS results for 1994/5 and for 2002/03 revealed that Australian Year 8 students' achievement in mathematics was significantly higher than the international average in all content areas considered. At Year 4 level, however, the picture was not quite as encouraging, with Australian Year 4 students performing at an overall level not significantly different to the international average. In the content areas considered, Australian students' mean score was significantly below the international average in Number, equivalent to the international average in Patterns and Relationships and above the international average in Measurement, Geometry and Data (Thomson & Fleming, 2004).

While such international assessment data can reveal important trends in student performance, there was no evidence from comments made by teachers and career professionals surveyed by Maths? Why Not? that these data are being used in an overt way to inform teaching and learning experiences or to improve school mathematics [The National Centre of Science, Information and Communication Technology, and Mathematics Education for Rural and Regional Australia (SiMERR), 25].

The results of both PISA and TIMSS suggest that our levels of achievement are sound overall, and the overall proportions of our students with low skills levels is not high compared with many other countries or the average spread of scores dissimilar from other similarly achieving countries. Nevertheless, they also show that there are far too many Australian students with low levels of skills and proficiency and large differences between the scores of the highest and lowest performers in Australia. Indigenous students, in particular, are overrepresented amongst those not achieving well, although it would be a mistake to conclude that the problems of innumeracy in Australia are confined to this group.

### **National numeracy benchmarks**

All Australian states and territories currently have testing programmes for numeracy for Years 3, 5 and 7 which can be used to monitor student achievement at school, system and state levels. These programmes are based on a national set of benchmarks against which states and territories report. The actual tests have not in the past been common although from 2008 they will be replaced by common national tests of numeracy in Years 3, 5, 7, and 9. The use of a common scale across Years 3, 5, 7 and 9 is intended to allow student growth in numeracy to be charted, and the use of secure equating tests to allow levels of achievement to be charted over time.

The tests came in for criticism from some submissions. Some authorities have asserted that their tests have a diagnostic role. Several submissions argued however that the test results are received too late to play this role, that the test items are not particularly helpful for this purpose or that the tests only assess particular aspects of the acquisition of mathematics rather than testing numerate behaviour. In the words of one submission:

Australian testing programme results need to be interpreted with some caution, however, as assessment of numeracy achievement in Australia has tended to focus on mathematical knowledge and, to some extent, strategic knowledge as it is difficult for pen-and-paper tests to authentically assess contextual aspects of numeracy knowledge [Queensland College of Teachers, 12].

Given that the current tests are not common and are not consistently on the same scale across year levels, comparisons between jurisdictions and between year levels are not particularly helpful. Nevertheless, in

general terms, the state and territory data suggest that approaching 95% of all children meet the Year 3 benchmark, around 90% the Year 5 benchmark and around 80% the Year 7 benchmark. It is interesting to note also that 'doing well' in respect to the assessments against the year 3 benchmark does not appear to be related to 'doing well' on the assessment against the Year 7 benchmark at the jurisdictional level although this may be a function of the lack of a common scale across year levels.

The national numeracy tests will provide information about the proportions of students who have achieved nationally agreed standards, but the interpretation of these standards is open to question. For example, the current benchmarks are criticised in some submissions as representing very low standards of attainment. Indeed, the benchmarks are explicitly defined as minimum standards without which a student would have difficulty progressing at school; they do not describe proficiency in numeracy, or even the minimum standards that the community expects from Australian schools. The existing Year 7 numeracy benchmark was set separately from the benchmarks for Years 3 and 5, and there is some evidence that it was set at a minimum desirable standard (Cooney, 2007), resulting in fewer students achieving the Year 7 benchmark than the earlier benchmarks. In contrast, the Key Performance Measure for PISA is the proportion of students who have achieved at the proficiency level or higher, and for TIMSS it is the proportion of students achieving above the test average. These proportions are therefore much lower than the proportions achieving the national benchmark standards.

## Excellence and equity

The evidence strongly indicates that improving literacy and numeracy performance requires a focus on both the quality and equity of student outcomes. With regard to the latter, one of the greatest challenges facing our education system today is how to improve the achievement of Australia's lowest performing students and schools, while also improving the achievement of the middle and highest performers.

McGaw (2007b) argued that there are three ways in which equity of performance in PISA should be looked at:

- 'the spread of scores, to determine how far poorly performing students are behind the high performers
- the relationship, at the individual level, between students' social backgrounds and their educational performances
- the relationship, at the school level, between differences in the social backgrounds of students enrolled and their average performances.'

## Overall range of performance

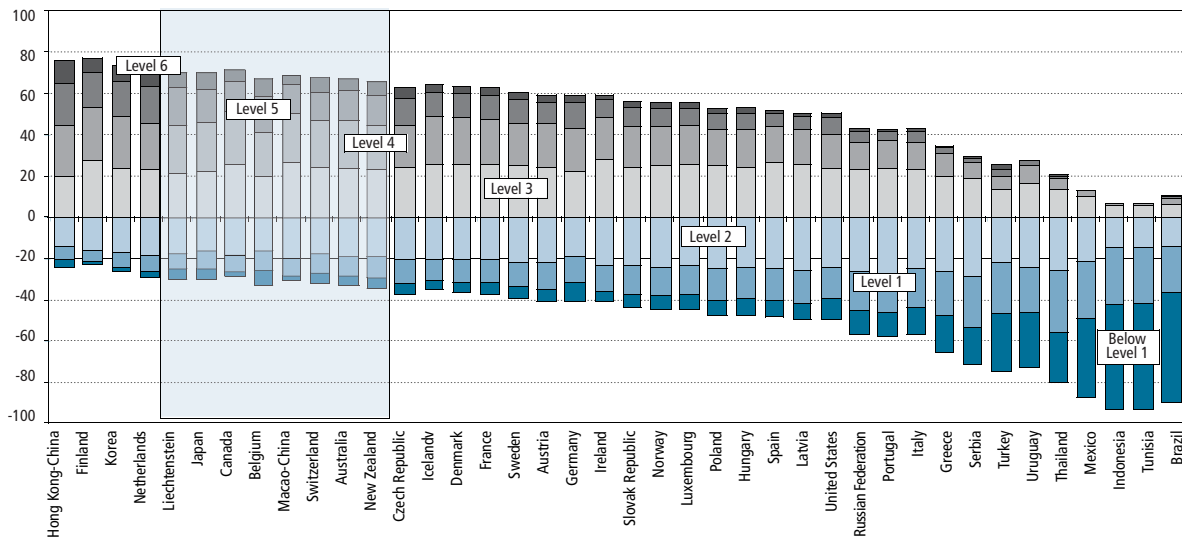
To examine the first of these, the distributions of students over the seven levels of performance defined on the mathematics scale in PISA 2003 are shown for the 40 participating countries in Figure 1. The countries with average results not significantly different from Australia's are shown in the shaded band with Australia.

The horizontal line at the zero point marks the difference between proficiency levels 2 and 3. This is regarded as representing a baseline level of mathematics proficiency on the PISA scale. The figure makes clear that the percentages of Australian students performing at the three lowest levels (0, 1 and 2) are not out of line with those in other similarly high performing countries, that is, Australia does not have a long 'tail' in mathematics or numeracy as is often claimed.<sup>1</sup>

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<sup>1</sup> In the Reading scale for PISA, there are slightly larger percentages of poorer performing students in Australia than in other similarly high-performing countries and this effect is often referred to as the 'long tail' in reading.

**Figure1 Performance at mathematics benchmarks, PISA 2003**



### Socio-economic background

To examine the second of McGaw’s points, relating to the relationship between socioeconomic background and performance at the individual level, the results on the mathematical literacy scale of PISA 2003 indicates that Australia’s performance in mathematical literacy follows the general pattern for the 41 OECD and other countries participating in the PISA 2003 survey. That is, students with lower levels of socioeconomic background score less well in the assessment. In Australia, ‘the relationship between socioeconomic background and mathematics performance is [a bit] less strong than the OECD on average’ (Thomson, Cresswell & de Bortoli, 2004, p.166) although this difference from the average is marginal.

On the third point, relating differences at the school level in the social backgrounds of students enrolled and their average performances, the great majority of Australian schools remain ‘comprehensive’ in the sense that students are not formally sorted into schools of different kinds on the basis of their previous academic performance, as is the case with countries such as Germany. Such streaming has the effect of reducing within school differences and increasing between school differences. Nevertheless, McGaw argues that the overall social background of the school has a stronger influence in Australian students than those in other countries with which we might wish to compare ourselves:

‘Our schools differ more markedly than those in the Scandinavian countries and, more significantly, 70% of the differences between our schools can be explained by differences in the social backgrounds of their students. That is, differences among Australian schools are much more influenced by whom they enrol than by what they do’ (McGaw, 2007b).

Marks’ (2006) analysis of PISA 2000 student data from 30 countries (including Australia) went beyond establishing relationships, instead indicating that socio-economic status (SES) could not substantially account for between- and within-school differences in students’ mathematics achievement. Rather, a factor for the allocation of students’ attainment within and amongst schools was perceived/measured ability. Marks used this model to attempt to explain the apparent paradox that when countries are considered one at a time, the degree of educational differentiation correlated with socio-economic inequalities in education.

Overall, in Australia, students from lower socioeconomic backgrounds perform on average substantially below students from higher socioeconomic backgrounds. Students from the lowest socioeconomic quartile in Australia are around eight times more likely than students from the highest socioeconomic quartile to have not achieved even very basic levels of proficiency.

Similar findings were apparent in TIMSS, with a clear positive (although only weak) relationship found between parental education and mathematics achievement, and between level of home educational resources and achievement. In TIMSS, principals also reported their estimates of the proportion of economically disadvantaged students in their school, and for both year levels there was better performance for the school overall in schools with few economically disadvantaged students.

### Indigenous students

Currently, Indigenous students lag behind the rest of the student population in terms of numeracy outcomes. In both PISA and TIMSS, Indigenous students achieved a mean score in mathematical literacy or mathematics substantially lower than that achieved by non-Indigenous students and well below the international averages in both studies. Specifically, in the PISA 2003 data, 'non-Indigenous Australians on average scored about one-quarter of a standard deviation above the OECD mean, Indigenous Australians more than half a standard deviation below the OECD mean. Clearly these differences are significant both statistically and educationally' (Thomson, Cresswell, & De Bortoli, 2004, p.85).

Comparisons with previous cycles of TIMSS indicates that the gap in achievement between Indigenous and non-Indigenous students has widened at Year 4 level largely due to a deterioration in performance amongst Indigenous students. Indigenous students are also substantially over-represented in the lowest proficiency levels and substantially under-represented in the highest proficiency levels.

The *2004 National Report to Parliament on Indigenous Education and Training* (DEST, 2006a) utilised student achievement data from 1999 to 2004 in the national numeracy benchmarks assessment for Years 3, 5, and 7 to point to Indigenous student gains. However, while the pattern of Indigenous achievement did mirror that of the rest of the students in Australia in the respective grade levels, the achievement gap between Indigenous and non-Indigenous students did not appear to narrow over the years during which initiatives targeted at Indigenous students' mathematics learning would have been executed. The dip in numeracy performance in Year 7 for Indigenous students was especially pronounced. Rothman's (2002) analysis of Australian 14-year-old students' mathematics scores indicated that in the period 1975 to 1998, differences between Indigenous students' and non-Indigenous students' mean scores remained statistically significant.

### Gender

Overall, there were no significant gender differences found in mathematics achievement in either year level in TIMSS 2002/03 or in PISA 2003. On closer examination, however, there were some significant differences in the domains tested. At Year 4 level, females outperformed males significantly in the area of geometry. At Year 8 level, all gender differences were in favour of males, and these were significant in the domains of number and measurement. In PISA, males significantly outperformed females in all domains; significantly so in the areas of space and shape and uncertainty. A similar proportion of males and females were found achieving below proficiency level 2, however a slightly higher proportion of males were achieving in the highest proficiency levels.



## Geographic location

In PISA, students attending schools in metropolitan areas performed better on average than students attending schools in provincial areas. Students in provincial areas in turn performed better than students attending schools in remote locations. All of these differences were statistically significant. Differences at both year levels were also evident in the TIMSS data, however the standard errors were large and so the differences were not statistically significant (Thomson, Cresswell & de Bortoli, 2004; Thomson & Fleming, 2004; Thomson, McKelvie, & Murnane, 2006).

This data is supported by the jurisdiction-based data collected in Australia to assess achievement against the national numeracy benchmarks. Analysis of the 2003 Year 3, 5 and 7 numeracy benchmark data by Pegg (2007, Table 1) found that at each year level, a significantly lower proportion of students in remote areas than their peers in metropolitan and provincial areas achieved the numeracy benchmark. Very remote students, in turn, achieved at a much lower level, with a significantly lower proportion achieving the numeracy benchmark than even those in remote areas. Disconcertingly, the gap between rural and remote Australian students and those in provincial or metropolitan areas becomes wider as students progress in school, possibly suggesting a stronger school(ing) effect than home/community effect.

**Table 1 Percentage of students achieving the numeracy benchmark, 2004 (with 95% confidence limits)**

	Metropolitan	Provincial	Remote	Very remote
Year 3	94 (1.1)	94 (1.4)	87 (3.7)	72 (6.2)
Year 5	92 (1.1)	91 (1.5)	82 (3.7)	59 (4.9)
Year 7	84 (0.8)	80 (1.1)	73 (3.4)	51 (4.9)

A number of issues identified by the SiMERR research programme *An Exceptional Schooling Outcomes Project* (AESOP) are relevant to rural and remote areas. Attracting and retaining experienced capable staff is a problem, so rural and remote schools largely miss out on a critical building block for achieving the outstanding outcomes identified in the research, that is, stable and experienced staff. As well, rural and remote schools are often small, and may only have one mathematics teacher. In these cases there is no sharing of resources possible within a school, nor a 'critical mass' of teachers to interact and share ideas. Professional development opportunities might also be difficult for those in remote or very remote areas [SiMERR, 25].

In summary, the evidence suggests that while overall levels of numeracy/mathematics achievement in Australia are quite good by international standards, there is an unacceptable proportion of Australian students (particularly but certainly not only amongst Indigenous students) who are not achieving acceptable levels of proficiency.

In Australia, targeted interventions tend to be directed at students identified as at risk of not meeting the National Benchmarks. These assess minimum standards rather than desirable levels of proficiency, the implication being that minimum standards are good enough, at least for some students. All students and their families, however, have a right to expect high quality, not minimum, numeracy outcomes from their schooling.



## 1.4 Participation in mathematics

This Review is taking place in the context of a commitment to enhancing workforce participation and a view that high levels of numeracy are a critical component of this. While the quality of the mathematical curriculum and pedagogy provided are critical in assuring high quality and high equity numeracy outcomes, the time available for learning and using mathematics is likely also to be a significant contributor to learning outcomes. This has two facets to it. How much time do students spend learning and using mathematics in the compulsory curriculum? How much and what mathematics do they study when it is no longer compulsory?

### Time on task

The Australian Primary Principals in their submission note that since the 1960's, despite mathematics being 'radically revised... time allocated...remained relatively constant' (p.1). Indeed the report, *Science and Mathematics in the Formative Years*, prepared for the Prime Ministers Science Council in 1990, pointed to

... the substantial problem of trying to teach more in less time ... [I]t may be suggested that certain aspects of traditional mathematics should be de-emphasised to allow new content or processes to come in. But ... if a particular procedure or fact is to be tested it has to be learned. De-emphasising simply means it has to be learned in less time and ... students 'learn' a lot badly, in the name of 'getting through the course' (Willis, 1990).

Not surprisingly, there is considerable variation between the amount of time allocated to mathematics on the timetable across schools and grade levels, with possibly even greater variation in the time in reality (see, for example, Clarke et al., 2002). The 'numeracy hour' has become common practice in primary schools in some jurisdictions (e.g. Victoria). AAMT in its submission argues that while it is difficult to be precise about average time allocations to mathematics in junior secondary schools, it is far less than the 300 minutes implied by the numeracy hour. AAMT data indicates that around 210 minutes may be the average. In its submission, AAMT notes the widespread belief that time on task for mathematics has diminished over the past 30 years, with no parallel decrease (and possibly an increase) in what is expected to be taught and learned. Given that the recently developed guidelines for school mathematics imply no reduction in content, action needs to be taken to ensure that an appropriate time allocation is present in the junior secondary years for mathematics.

The Australian data is not dissimilar to the average across the countries participating in TIMMS, that is, 250 minutes in Year 4, and 210 minutes in Year 8, however, if Australia aspires to be one of the very high performing countries it has to decide what investment it is prepared to make and what it should prioritise.

### Recommendation 2:

That all jurisdictions should work towards a minimum of 5 hours per week of mathematics for students in all the primary Years K-6/7 and a minimum of 4 hours per week in all the lower secondary Years 7/8-10. This time should include cross curricular learning.

## Participation at senior secondary level

The Australian Mathematical Sciences Institute (AMSI) points to the necessity for a good grounding in mathematics for all students:

Without a reasonable level of competence in mathematics, good skills in trades such as plumbing, electrical, carpentry and building are hard to achieve. Biology and medicine are being transformed by the genomic revolution, which is underpinned by statistics, dynamical systems and many other areas of mathematics. Economics and commerce and engineering are areas that are not accessible to students without mathematics at Years 11 and 12. Jobs growth in the Australian economy during the recent economic boom has been mainly in the professional and management areas, requiring university level skills. Without considerable mathematical knowledge, many professions are not available to students [AMSI, 49].

Nevertheless, since 1995 participation in Year 12 advanced and intermediate level mathematics courses has fallen dramatically and recent reports show it continues to fall. Arguably, one of the contributing factors to this is that universities have dropped advanced mathematics as a prerequisite for many courses. There are a range of other factors that need to be considered, however, including lack of preparedness for these courses, poor advice in schools concerning the role of mathematics in many career choices, the Year 11 and 12 courses offered, lack of appropriately qualified teachers and assessment and scaling systems that fail to adequately reward students for taking harder subjects (Barrington, 2006; Fullarton, Walker, Ainley & Hillman, 2003) [AMSI, 49].

The *Maths? Why Not?* project directly investigated the question: Why are capable students not choosing to take higher-level mathematics in the senior years of schooling? Both teachers and career professionals who responded to their surveys were in agreement concerning the four most important influences affecting students' engagement with mathematics. These were:

- self-perception of ability
- interest and liking of mathematics
- previous achievement in mathematics
- the perceived difficulty of mathematics.

These were all student-related influences, with self-perception of ability given the highest ranking by both groups. This suggests that the nature of feedback to students about their performance both in mathematics and relative to other subject areas is vital. Teachers need to focus on meaningful feedback to students, in the form of both formative and summative assessment, as part of providing students with a realistic view of their abilities. Such feedback needs to be coupled with 'relevant and enjoyable teaching and learning strategies delivered by appropriately educated teachers at each stage of schooling in order to ensure that students experience regular success' [SiMERR, 25].

A number of recommendations are being developed as part of the *Maths? Why Not?* Project. Key points underpinning the recommendations bearing on this review include:

- analyse PISA and TIMSS data concerning attitudinal characteristics of students from countries that are more successful than Australia
- identify the characteristics of early learning experiences that promote understanding and interest in mathematics
- research the conceptual obstacles experienced by students in the middle years of schooling

- research the role of formative and summative assessment procedures in early secondary mathematics and their effect(s) on student self-efficacy
- research problematic components of the curriculum and teaching
- develop 'second-chance' programmes that offer junior secondary students opportunities to consolidate their understanding at critical developmental points in their learning
- develop learning units that explore and illuminate links between careers and mathematics
- establish incentives to encourage mathematics graduates into primary and secondary mathematics teaching.

Stacey (2004) provided evidence from surveys which found that teachers of mathematics and career professionals perceived that students' decision making was strongly influenced by the appeal of less demanding subjects, particularly in terms of the sustained effort required to succeed in mathematics. The most pertinent comments related to comparisons made with other subjects in the areas of real-world interest and levels of achievement. 'Other subjects' are generally regarded as having a greater real-world interest for students than mathematics and, therefore, mathematics struggles to compete.

An issue of parity of achievement also emerged with teachers in particular pointing out the need for students to be given a sense of the level of their achievements in a subject, such as mathematics, which is perceived to be hard in comparison with other subjects undertaken. The flow-on effect can be that students perceive that they are not achieving appropriate results and this sets up a negative view of the ability and potential to pursue mathematics.

Data from the *Longitudinal Surveys of Australian Youth* indicate that gender differences in participation in mathematics courses are marked. Of those studying mathematics in this final year, 54 per cent of the enrolments in advanced mathematics were males, compared to 42 per cent of those enrolled in fundamental mathematics courses. Multivariate analysis found that the odds ratio for advanced mathematics for females to males was 0.54, meaning that, all other things equal, the likelihood of a female enrolling in advanced mathematics at Year 12 is much less than that for a male (Fullarton, Walker, Ainley, & Hillman, 2003).

### **Enrolments in tertiary mathematics courses**

With respect to levels of participation in mathematics, industry, business and the higher education sector in Australia have flagged an emerging shortage of qualified mathematicians and statisticians. The *Australian Council of Deans of Science Report* (Harris & Jenz, 2006) and the *National Strategic Review of Mathematical Sciences Research in Australia* (Australian Academy of Science, 2006) each urge a greater emphasis on the preparation of mathematicians, such preparation involving all levels of education.

Dobson's (2007) report to the Australian Council of Deans of Science shows the decline in participation in mathematics and statistics in the universities. The report documents that:

In 1989 there were 7,520 equivalent full time science students enrolled in mathematics; in 2005 this number had dropped to 4,988. This is a decline of 2,532 equivalent full time students, or about one third (Dobson, 2007, p.71).

This is a world-wide phenomenon – the literature from the UK and the US, as well as other countries, report similar concerns. In Australia, however, the situation may have reached a more critical point. In 2003 OECD figures showed that only 0.4% of Australian university students graduated with qualifications in mathematics or statistics, compared with the OECD average of 1% [AMSI, 49].

The *Science, Engineering and Technology (SET) Skills Audit* in August 2004 raised a number of issues around uptake of science, engineering and technology studies and career paths related to:

- some skills shortages in many engineering and some science disciplines
- a static or declining proportion in the enabling sciences and advanced mathematics in schools and in post-school settings
- concerns about the quality of science education
- teacher education in Australia is a large and diverse enterprise
- the increasing need for SET graduates to have enabling and cross-disciplinary skills (DEST, 2006b) [DEST, 28].

In summary, the evidence on participation in mathematics suggests that many students lack confidence in the subject, do not enjoy or see personal relevance in it and are unlikely to continue its study voluntarily. This clearly is a risk to Australia achieving its human capital goals, but the personal and social consequences for individuals and their families and communities can be unfortunate in ways that go beyond the purely economic.

## 1.5 The teaching workforce

The main concerns associated with Australia's mathematics and numeracy teaching workforce are that:

- primary teachers are not being adequately prepared for teaching numeracy and mathematics
- there are insufficient numbers of qualified secondary and post secondary mathematics teachers to meet Australia's education needs
- not all teachers who currently teach mathematics have appropriate expertise in these areas.

More detailed data on mathematics teachers will be available from the *Staff in Australia's Schools Survey*, which is being conducted by the ACER over 2006–07. This survey is intended to obtain data relating to teacher workforce needs and address key gaps in the data available to characterise the profession. It will also investigate and provide advice on longer-term approaches to data gathering to support teacher workforce planning.

### Primary teachers

Concerns are often expressed about levels of mathematics content knowledge of Australian primary teachers, and a variety of in-service programmes have been initiated in order to tackle this problem. In Victoria, the guidelines for the Victorian Institute of Teaching state that primary teachers are expected to have good skills in numeracy. The guidelines specify that a desirable target is at least the satisfactory completion of Victorian Certificate of Education (VCE) Mathematics Units 1 and 2 [Victorian Department of Education and Victorian Curriculum and Assessment Authority, 48].

The *Prepared to Teach* report (Louden, Rohl, Gore, Greaves, McIntosh, Wright, Siemon & House, 2005) found that almost all pre-service primary teachers felt they had personal numeracy skills adequate for teaching, and about two-thirds of the senior staff in the schools agreed with this assessment. More than three-quarters of these pre-service teachers believed they were well prepared in each of the content areas.

In contrast however, during 2005, the Education and Training Committee of the Parliament of Victoria conducted an *Inquiry into the Promotion of Mathematics and Science Education* which noted that teacher

quality was a significant factor in the effective learning of mathematics with concerns being expressed from stakeholders about primary school teachers' level of knowledge and conceptual understanding of mathematics. [Victorian Department of Education and Victorian Curriculum and Assessment Authority, 48]. This suggests that while pre-service primary teachers may consider themselves to have the necessary numeracy skills for teaching mathematics (supported by their senior colleagues), other stakeholders are not so sure. Stakeholders it seems, may hold alternate perspectives on the required level of knowledge and conceptual understanding for effective mathematics teaching.

## Secondary teachers of mathematics

The AAMT argued in their submission that a lack of suitably qualified teachers of mathematics limits a school's capacity to provide effective programmes for all students. Shortages of qualified mathematics teachers may result in a decrease in time spent on mathematics, or mathematics teaching by those who are teaching out of their field.

As AAMT pointed out, and is widely acknowledged, retraining courses in many jurisdictions (e.g. South Australia, New South Wales, and Western Australia) is evidence of the shortage. These programmes tend to be expensive, relatively short duration courses for existing teachers who have generally studied insufficient mathematics in their formal education. Programmes of incentives to attract suitable qualified graduates to mathematics teaching are also prominent (e.g. Tasmania, South Australia) [AAMT, 31].

There is anecdotal evidence, and data provided informally by systems, to suggest that a significant proportion of teachers of secondary mathematics do not have expertise in the field. The Federation of Australian Scientific and Technological Societies reports 'it is estimated that about 40% of junior secondary students are taught mathematics by a teacher who has little or no background in mathematics and no studies in the teaching of mathematics.' (Thomas, 2000, p.2) This latter statement is supported by data from TIMSS 2002, in which around 30% of Australian teachers of Year 8 students surveyed had neither mathematics or mathematics-education as their major area of study (Thomson & Fleming, 2004).

There are two main categories of 'out of field' mathematics and numeracy teachers:

- teachers of other subject areas who only occasionally teach mathematics or teach only a class or so, possibly as their second or third 'string' or to 'help out', and who are unlikely to identify themselves as 'mathematics teachers'. Typically such teachers endeavour to do their job without having content or pedagogical expertise in mathematics. When undertaking professional development, they are likely to favour their 'main' teaching areas.
- teachers originally from other subject areas or primary teachers who now identify as mathematics teachers but lack qualifications which provide content expertise. They may have had some professional development to upskill particularly in pedagogy, although less commonly in content.

Once these teachers are in place, it appears that only occasionally is anything done to address their special development needs. A highly committed and enthusiastic teacher of Art or Literature who can inspire a new generation to their field, may have difficulty doing the same for mathematics no matter how conscientious or committed they are to their students. They are likely to lack the depth of knowledge, the extent of examples, and the excitement it takes to teach really well.

## Teachers of numeracy across the curriculum

The *Prepared to Teach* report (Louden et al., 2005) found that only three-quarters of beginning secondary teachers in their sample rated their personal numeracy skills as adequate for teaching general numeracy (as distinct from the subject mathematics per se). Indeed, 23% of beginning teachers felt adequately prepared to teach Algebra, 33% Chance and Data, 36% Space, 37% Number and 38% Measurement. More than half of the senior staff reported that beginning teachers were adequately prepared to teach number, measurement and space, but they were less convinced about chance and data and algebra (Louden et al., 2005).

Some studies have indicated that there is little specific provision in Australian schools to develop teacher knowledge for teaching numeracy to students with either learning difficulties (Louden et al., 2005) or disabilities (Van Kraayenoord, Elkins, Palmer, & Rickards, 2000). 'If teachers beginning their careers in schools feel unprepared to teach these students it is likely that this lack of provision will continue' (Louden et al., 2005, p.110). The majority of teachers in the *Prepared to Teach* report, however, did feel adequately prepared for teaching numeracy to diverse groups of students, including those with learning difficulties.

## Technical and Further Education (TAFE)

An area that was not addressed to any extent in the submissions received was TAFE education. Thomas (2000, p.22) argues that "(M)athematics has little status in the TAFE and community education sector." Mathematics tends to be a 'service' subject in TAFE, and often taught by sessional and contract staff. As with school mathematics teaching, there is little data on who is teaching mathematics and with what qualifications they are doing so. However there is a diversity of needs in TAFE, and so the TAFE sector should be able to both support good mathematics teaching and provide a second chance for the young people and adults whose schooling has failed them. Thomas suggests that 'the shortage of mathematics teachers in the secondary sector has implications for TAFE. This should be addressed by the TAFE sector taking more responsibility for the education of its teachers' (p.22).

## University

Currently there are insufficient data to make an informed commentary on university teaching of mathematics. The majority of secondary teachers are taught in mathematics and science faculties. Whilst one submission [ATN, 6] questioned whether the mathematics taught in these faculties is relevant to those students preparing to be engineers for example, not enough is known currently about the adequacy of mathematics teaching at university for those students who will in the future teach mathematics whether in secondary schools or in universities.

In summary, the quality and commitment of the mathematics and numeracy teaching workforce, at all levels of education, is critical if Australia is to achieve its objectives and to improve workforce participation and productivity. While it is often suggested that too many primary teachers lack desirable levels of mathematical competence and confidence, in secondary schools there are a great many teachers teaching mathematics 'out of field'. They have often had no preparation or only a short term 'retraining' programme. This will not change in the medium term and significant professional development is needed so that the next generation of students is not disadvantaged. Many others, teaching in fields other than mathematics, may be ill-equipped and/or disinclined to address the demands of numeracy 'across the curriculum'.





# Chapter 2: Supporting mathematics learning

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## 2.1 Introduction

This chapter reviews and analyses relevant submissions to this review and recent national and international mathematics education research into the practices and norms in the school mathematics classroom, with an emphasis on documenting evidence relating to the effectiveness of existing programmes, policies and projects that have been initiated to facilitate, sustain and/or further improve mathematics outcomes both within Australia and internationally.

It also examines studies and submissions related to ways of supporting students' numeracy learning. It examines aspects of teaching, including characteristics of effective teaching, the role of language in numeracy learning, and classroom based assessment strategies. It presents a perspective on grouping by 'ability', and approaches to fostering positive student motivation. It considers approaches to teaching adult learners, the role of technology overall, and looks at out-of-school support such as parents and coaching classes.

International evidence regarding mathematics and numeracy education was systematically reviewed for this report. This involved reviewing and synthesising three types of meta-analytic papers; a) Statistical meta-analyses<sup>2</sup>, b) Systematic Reviews<sup>3</sup> and c) Best Evidence Syntheses<sup>4</sup> (Appendix 2). Together the statistical meta-analysis papers cover 462 research studies with quantifiable and comparable research effects. The Systematic Reviews and Best Evidence Syntheses findings are based on a total of 946 research studies. In general, the international studies included focused on evaluation of mathematics programmes or interventions. Several themes emerged from this meta-analysis.

## 2.2 International evidence of good practice

Internationally, Finland has been held up as the shining light in education since its students topped the OECD in reading in PISA 2000. Finnish students maintained this high standard, ranking as one of the highest performing countries in mathematical literacy. Many studies have since examined what it is that Finland does so well in education.

The Finnish school system is a flexible, decentralised system with a large portion of the educational decisions made at the local, or even school level. There are very few private schools in Finland, and these rarely charge for tuition. Teaching is highly regarded within the local community, and both primary and secondary teachers take a master's degree in pedagogy or in one or two teaching subjects. In-service education has a strong emphasis in the Finnish education system. There are a wide variety of organisations that provide in-service courses for primary and secondary school teachers, and each university has a centre for continuing education. Collaboration is encouraged to ensure continuity of curriculum from pre-school onwards.

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- 2 Statistical meta-analyses involve the combining, or pooling, of effect size calculations from individual research studies addressing a common research issue.
  - 3 Systematic Reviews are large reviews of research evidence on a particular topic. A comprehensive search is followed by exclusion/excluding papers inappropriate to the review question and then grading according to research quality. High quality studies are retained and synthesised. In some cases a statistical meta-analysis is also performed to pool effect sizes.
  - 4 Best Evidence Syntheses are large scale inclusive reviews of research evidence on a particular topic. They are similar to Systematic Reviews but differ on some methodological points and reporting style as they have a strong orientation toward local practitioner readers.



Part of Finland's success has been attributed to the LUMA project. LUMA was developed by the Finnish National Board of Education, and was run between 1996 and 2002 (Valijarvi et al, 2002). The project introduced new approaches to the teaching of mathematics to a large number of teachers, many of whom were underqualified in the teaching of mathematics. LUMA brought together researchers from universities, school teaching staff, civil servants in educational administration, and professionals working in industry. Its emphasis was on experimental thinking, problem-solving and the use and application of knowledge in mathematics.

Mathematics education in the UK has experienced many similar challenges to Australia and the report from the committee chaired by Professor Adrian Smith examined many of the same issues.

Smith (2004) identified three key areas for concern:

- 'First, we have a curriculum and qualifications framework that fails to meet the mathematical requirements of learners, fails to meet the needs and expectations of higher education and employers and fails to motivate and encourage sufficient numbers of young people to continue with the study of mathematics post-16.
- Secondly, we have a serious shortage of specialist mathematics teachers in schools and colleges and this is having an adverse effect on pupils' learning experiences.
- Thirdly, there is a lack of support infrastructure, both at national and local levels, to provide continuing professional development and resources, including Information and Communication Technologies (ICT), in support of excellence in the teaching and learning of mathematics' (Smith, 2004).

These areas will seem all too familiar to Australian mathematics educators. Some of the findings and recommendations include designing new pathways for mathematics, reworking the curriculum, providing teachers of mathematics with greatly enhanced resources and sustained access to professional support and development. Smith (2004) also provides a recommendation that a National Centre for Excellence in Mathematics Teaching, together with nine Regional Mathematics Centres be established to provide Continuing Professional Development for teachers.

After the release of results for TIMSS 1995, in which US performance was poor, funding was made available from the US for a limited TIMSS 1995 video study. This initial study was successful, and from it was developed the TIMSS 1999 Video Study, which examined classroom teaching practices through in-depth analysis of videotapes of eighth-grade lessons in mathematics and science. The mathematics portion of the TIMSS 1999 Video Study included 638 eighth-grade lessons collected from all seven participating countries, and provides rich descriptions of mathematics and science teaching as it was actually experienced by eighth-grade students in seven countries.

In addition to the US, participating countries included Australia, the Czech Republic, Hong Kong SAR, Japan, the Netherlands, and Switzerland. Students in these countries were generally among the top-performing students on the TIMSS 1995 mathematics assessment and, in particular, outperformed their US counterparts. By studying nationally-representative samples of eighth-grade mathematics and science lessons, the TIMSS 1999 Video Study provides educators and policymakers a better understanding of how national, regional, and local policies related to curriculum and instruction are being implemented in the classroom.

The *Learner's Perspective Study* (Clarke, Keitel & Shimizu, 2006), also intensively studied international mathematics classrooms and noted effective strategies used by competent teachers. According to Clarke et al. (2006), competent teachers around the world have developed very different approaches to many similar problems: how to begin the lesson and how to conclude it; what tasks to pose and when and how; and how to monitor and assist student learning. Common events in one classroom are novel in another and some of our most entrenched assumptions are challenged by the practices in other countries.

A finding from the *Learner's Perspective Study* is that terms such as 'teacher-centred' and 'student-centred' are entirely misleading when applied to classrooms in China, and the balance between speaking and listening is quite different for both teachers and students depending on cultural context.

The study found that good teachers are innovative; that they recognise that there are many different ways of doing things and that they need a variety of 'tools in their toolbox'. The study also found other key differences in teaching and learning in classrooms around the world. It is important to note that the numbers of teachers involved from each country were small and generalisation cannot be justified. The evident differences were interesting nevertheless in illustrating the variation, including:

- **China** – where cultural norms mean that student engagement and participation in class discussion must be cleverly orchestrated by teacher observation and invitation.
- The **Philippines** – where class sizes of 60 to 80 students mean that a competent teacher must develop novel motivational and organisational strategies to maintain student interest and participation.
- **Japan** – where teachers employ a sophisticated professional vocabulary to discuss, develop and refine what happens in their classrooms.
- **South Africa** – where mathematics is integrated into a thematic curriculum focusing on societal issues such as HIV-AIDS and substance abuse.
- **Singapore** – where the instructional strategies developed to produce high student test performance seem in tension with those required for the development of creativity and problem solving expertise.
- **Sweden** – where the teacher strives to demonstrate the relevance of mathematics, but the students don't quite buy the message.
- **Australia** – where the competent teacher's commitment to interacting with every student every lesson has led to quite distinctive instructional practices.

Clarke (2006) explained that the differences in classroom practice at the heart of the Learners' Perspective Study should 'be interpreted as local solutions to classroom situations and, as such, be viewed as complementary rather than necessarily oppositional alternatives, within a broadly international pedagogy, from which teachers in different countries might choose to draw in light of local contingencies' (p.376).

## 2.3 Teachers and teaching

This section outlines some common ways of thinking about teaching and teacher actions, and summarises some characteristics of effective teachers.

### Perspectives on teaching

Various perspectives guide thinking about teaching. One perspective is a social constructivist view that emphasises the importance of students having opportunity to 'create' mathematics concepts and link them to existing concepts for themselves.

Another perspective is termed *socio-cultural*, a key aspect of which is Vygotsky's (1978) Zone of Proximal Development (ZPD) which he described as the 'distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by problem solving under adult guidance or in collaboration with more capable peers' (p.86). ZPD defines the work of the class as going beyond tasks or problems that students can solve independently, so that the students are working on challenges for which they need support. It also provides a metaphor for the support that teachers can give to students experiencing difficulty.

A further perspective on teacher actions is based on the work of Cobb and his colleagues (e.g. Cobb & McClain, 2001), involving two complementary norms of activity in mathematics classrooms which they describe as mathematical norms and socio-mathematical norms. The *socio-mathematical norms* include the usual practices, organisational routines, and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, and responsibilities of individual learners. The *mathematical norms* are the principles, generalisations, processes, and products that form the basis of the mathematics curriculum and serve as the tools for other learning.

Sullivan, Zevenbergen and Mousley (2005) extended the notion of mathematical norms and use the term *mathematical community norms* to encompass not only 'classroom actions and interactions that are specifically mathematical' (Cobb & McClain, 2001, p.219) but also norms of practice. In particular, their conceptualisation includes elements such as culture, social group, language comprehension and usage, and classroom organisation as they relate to the teaching and learning of mathematics.

These perspectives provide a framework for different aspects of the task of mathematics teaching overall.

## Characteristics of effective teaching

Slavin and colleagues in the US recently conducted two Best Evidence Syntheses to examine the relative effectiveness of different types of mathematics programmes in elementary (Slavin & Lake, 2007) and in middle and high school mathematics (Slavin, Lake & Groff, 2007). The studies they examined fell into three broad categories, i) mathematics curricula programmes, primarily text-book based; ii) supplementary Computer Aided Instruction; and iii) instructional process or classroom practice programmes. Across elementary to high school mathematics they found the instructional programmes to be most effective in lifting mathematics attainment. They also concluded that research on instructional process tended to be of a much higher quality than the research on mathematics curricula and computer aided instruction.

Instructional programmes based around cooperative learning, like the Classwide Peer Tutoring programme, showed the largest gains in student learning. Cooperative and collaborative learning approaches, including peer-tutoring programmes, were also shown to be effective in other statistical meta-analyses (Haas, 2005; Springer, Stanne & Donovan, 1997). In particular, these approaches were effective with low attaining 'at risk' students and students with special educational needs (Baker, Gersten & Lee, 2002; Kroesbergen & Van Luit, 2003; Kunsch, Jitendra & Sood; 2007).

Other instructional approaches with more modest effects included mastery learning and direct instruction approaches (Slavin & Lake, 2007; Slavin et al., 2007). Direct instruction has been described as a 'systematic method for presenting learning material in small steps, pausing to check for student understanding, and eliciting active and successful participation from all students' (Rosenshine 1986, p.60).

In meta-analyses of instructional approaches that are specifically effective for secondary algebra, direct instruction was shown to be the most successful, however, collaborative learning also showed moderate positive effects. Problem based learning had a large positive affect on pre-algebra classes, but a small negative effect upon algebra classes particularly for high ability students (Haas, 2005). In a review of research in special education settings, Purdie and Ellis (2005) found that direct instruction strategies consistently yielded large positive effects on students' learning and achievement progress. However, a review in preparation for the Background Paper for this Review indicated that there does not appear to be conclusive evidence that direct instruction in the mathematics classroom leads to enhanced mathematics outcomes. Certainly, there was some disagreement in the submissions about the evidence for and appropriateness of direct instruction for preparing children today [see The Australian Catholic University, Victoria, 14; Australian Association of Special Education, New South Wales, 19; Rowe, 47]. Part of the problem may be that the label is interpreted and understood differently. Brown et al. (1998), for example, highlighted how what they described as direct interactive teaching, meaning teacher-led instruction, was understood differently by teachers participating in the National Numeracy Strategy in Britain: instead of direct interactive teaching incorporating high-level questioning from the teachers, this core value of the teaching approach was lost in an initiative which emphasised fast-paced lessons.

At times the term is conflated with explicit instruction and at other times distinguished. The very recent review by Hiebert and Grouws (2007), two leading world scholars in mathematics education, is helpful here. In their summary, they note, 'we believe that two features of classroom teaching facilitate students' conceptual development (and perhaps mathematical proficiency), explicit attention to connections among ideas, facts, and procedures, and engagement of students in struggling with mathematics' (p.391).

The focus on classroom instructional practice was a dominant theme throughout the meta-analytic reports and there is broad support for approaches that employ cooperative, collaborative, dialogic strategies in teacher-led, peer and individually driven learning. One of the strongest conclusions of the New Zealand Ministry of Education Best Evidence Synthesis is that successful mathematics teaching and learning is focused around the development of a positive and culturally appropriate student mathematical identity (Anthony & Walshaw, in press). This perspective is also strongly supported by the evidence relating to student motivation in mathematics, which places development of students' identity as mathematics learners as central to motivational effort (Kyriacou, Golding & E.M.E.R., 2006).

The British study, *Effective Teachers of Numeracy* (Askew, Brown, Rhodes, Johnson, & William, 1997) found that relatively high mean achievement gains were not necessarily related to overall teaching style. Instead, effective teachers tended to be those who:

- had 'connectionist' orientations (as opposed to 'transmission' or 'discovery' orientations)
- focused on students' mathematical learning (rather than on provision of pleasant classroom experiences)
- provided a challenging curriculum (rather than a comforting experience)
- held high expectations of initially low-attaining students.

In Australia, Beswick's (2007) research with 25 secondary school teachers and their mathematics classes in Tasmania also revealed that teacher beliefs which were aligned with constructivist ideas did not associate with any particular pedagogic strategy. In other words, classroom learning environments are regulated by the teacher beliefs and principles underpinning whatever pedagogic methods which might have been employed to realise these very beliefs and principles.

Many of the effective teacher features listed in the previous set of dot points constitute meaningful and constructive classroom interactions between teacher and students, and perhaps also amongst students. Indeed, students connecting with the subject and with peers and teachers is one of the characteristics identified as making the difference to numeracy attainment within classrooms in a large-scale, 65-school project in New South Wales, entitled *What's 'Making the Difference' in Achieving Outstanding Primary School Learning Outcomes in Numeracy?: Strategic Numeracy Research and Development Project* (Busatto, 2001).

Brown et al. (2001) concluded that 'although there is some evidence that certain behaviours are effective in teaching mathematics their effect seems to be small and variable' (p.16). Nevertheless, the kinds of lists above inform those with responsibility for aspects of teacher professional learning, and teachers themselves.

While the strength of any one variable might not be established, that teachers make a difference is evident in a number of studies. For example, in a detailed study using data from the Early Numeracy Research Project in Victoria, Sullivan and McDonough (2002) noted that:

teachers who were given extensive professional development ... differed substantially in the extent to which their students improved in defined growth points ... the differences between the most effective and least effective teachers are substantial. The teachers who were effective seemed able to articulate focused, developmentally appropriate and engaging activities for their students, and engage them actively in interrogating those experiences (p.255).

While there are particular characteristics associated with more effective teachers, formulating an adequate and quantifiable definition of quality teaching is challenging – what works in one context may not work in another. Measuring the extent to which teachers have caused student gains on assessments is also difficult. This suggests that care should be taken in developing measures of quality teaching.

An extensive review of numeracy strategies employed in Australian states and territories was completed by Doig, McCrae & Rowe (2002). Following the identification of a number of effective strategies identified by the literature, *Project Good Start* sought to investigate the practices and learning experiences that support the early numeracy development of children in the year before school and the first year of schooling. This study found, *inter alia*, that curriculum in the early years was poorly defined, and that teachers rarely had access to professional development in the area of numeracy (Thomson, Rowe, Underwood & Peck, 2005).

There seems to be particular concerns about aspects of mathematics teaching in Australian schools, especially in the middle years. Stacey (2003), for example, compared the Australian average Year 8 mathematics lesson with those of the high-achieving countries in the TIMSS 1999 Video Study. Stacey argued that the characteristics of teaching evident constituted what she called the 'shallow teaching syndrome' in a typical Australian middle school mathematics lesson. The Australian data registered the highest national percentage of repetitive problems, lowest percentage of problems which were 'mathematically related' (that is, where a problem would lead the student to extend particular method(s) used in a previous problem), and highest percentage of problems of low complexity. As with the Netherlands and the US, in Australia there was very little evidence of lessons involving mathematical reasoning.

As Stacey argued, Australian 'curriculum documents of the last decades show a reduced emphasis on computational skill and algebraic procedures, and substantial emphasis on students obtaining deep understanding of the underlying ideas and being able to use them in real contexts' (p.120). She went on to conclude that the Australian data from the TIMSS 1999 Video Study pointed to a 'less-than-expected performance on conceptual understanding and problem solving ability' (p.121), and that the greater emphasis on the fostering of conceptual understanding and problem-solving capability did not appear to have given Australia the benefits from the trade off of routine skills. This is, of course, assuming that the *implemented* curriculum is similar to the *intended* curriculum. The *Connected Mathematics Programme* in the US over the years 1991 to 1997 advocated for middle school mathematics to be taught in ways where students learn in groups through investigations and exploration. Over the project years, the 1250 students made real progress in their performance.

### **Recommendation 3:**

That from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking.

There are concerns in the literature about what appears to be conventional wisdom about teaching. For example, the common belief that teaching aids (or manipulatives, or concrete materials) develop students' conceptual understanding and other aspects of mathematics learning both within Australia (e.g. O'Toole, 2006; Perry & Howard, 1994) and internationally (e.g. Szendrei, 1996) has more recently been challenged. Studies by Hart (1989) and Bobis (1993) are two examples of studies which questioned the notion that concrete materials add a sense of reality to the concept being learnt. While comments from classroom teachers indicate that their students enjoyed the hands-on activities, Bobis' study (1993) raised the concern that such materials lead to misunderstanding which can have far-reaching implications for students' mathematics learning. There is also the possibility that the deployment of multiple types of teaching aids contributes to cognitive confusion, instead of (paradoxically) promoting connections between and amongst related ideas. For example, Boulton-Lewis's (2000) longitudinal research with children as they progressed from Years 1 to 3 revealed that many of them became progressively incapable of representing numbers efficiently using the multi-based attribute blocks, when these children had been encouraged by their teachers over the three years to make use of different types of teaching aids to represent and manipulate numbers. Boulton-Lewis felt that the use of different materials had actually confused the children at a time when they were developing their mathematical concepts.

Mathematics textbooks are often prescribed in secondary school mathematics classes, and while most primary school students need not buy mathematics textbooks, this form of resource is often used to guide teaching and to facilitate learning, or as a source of worksheets (Groves, Mousley & Forgasz, 2006). Mathematics textbook publishing in Australia is not a regulated business, and schools are given the responsibility to select and appoint official textbooks, even if the selected titles might be from interstate and written to an interstate curriculum framework. Despite the widespread use of textbooks, there has been limited research into the preparation, content and use of such a material resource.

There is further scope for research both into ideal approaches to mathematics teaching and into approaches found commonly in Australian classrooms.



## 2.4 The role of language in mathematics learning

The decade or so before the turn of this millennium saw substantial research into how students' language abilities might impact on mathematics learning (see, e.g. Ellerton, Clements, and Clarkson, 2000).

Rothman (2002) examined the achievement data of 14-year-old Australian students across the years 1975 to 1998, concluding that while mean scores for mathematics of students from homes where the main spoken language was English remained higher than students who spoke other languages at home, the gap in these mathematics mean scores was narrowing. However, over the last few years, research into language factors in mathematics education has been rather sparse. In fact, Zevenbergen (2000) concluded that there had been little knowledge in any systematic way of the impact of language on the numeracy growth of primary school students. Similarly, there is limited research being conducted to assess or evaluate related programmes in the classroom.

Frequently the term 'literacy and numeracy' is used in educational contexts as a singular rather than collective term. For example, education systems and governments use the phrase in describing the shared and agreed foundational capacities for what is important for every student to attain through their schooling experiences.

The Australian National Literacy and Numeracy Plan clearly recognises the importance of both literacy and numeracy as the cornerstones of education. Yet, less often is the inter-relationship between the two domains examined despite the clear implications of language and literacy issues associated with the learning of mathematics.

What is frequently not considered is *mathematical language* as being distinct from mathematical literacy. For many children, mathematics is seen as a 'foreign language'; the symbols and expressions provide a formidable barrier to understanding of mathematical concepts. Teachers need to use these in situations and contexts which make their meanings clear and students should be given opportunities to practise the use of the language by reading and explaining them both orally and in writing. (*A National Statement on Mathematics for Australian Schools*, Australian Education Council & Curriculum Corporation, 1991 p.20). For example, if students were explicitly taught to read  $24 \div 6$  as 'twenty four divided by six' they might not attempt to compute the operation by dividing 24 into 6. Similarly, if  $\frac{3}{4}$  were initially read as 'three out of four equal parts' or if  $6f$  were initially read by students as 'six lots of whatever number  $f$  represents,' this language would support their learning of the concepts of fraction and variable respectively.

Adding to the language challenges children face in mathematics classrooms is the use of everyday English terms that have different meanings in mathematics classrooms. Words such as 'big', in the context of 'which number is bigger?' can be confusing because they refer to *amount* as opposed to physical size. Then there are verbs such as 'evaluate,' 'simplify' and 'factorise' which are infrequently used beyond mathematics lessons. All of these need to be explicitly taught within the mathematics context so that students are able to make sense of this 'language of mathematics'.

It is clear that competence in literacy underlies all facets of education, particularly in the early years. Anecdotally, teachers report that the Year 3 state/territory assessment instruments for mathematics require a Year 5 level of literacy. In a recent study that examined (*inter alia*) the relationship between literacy and numeracy outcomes for a national sample of 3,633 Year 1 to Year 9 students, it was found that reading achievement was the strongest predictor of mathematics achievement for both males and females (Rowe, 2006; Rowe, Stephanou and Hoad, 2007; Rowe, Stephanou & Urbach, 2006). PISA 2003 data provided similar findings. Reading literacy amongst 15-year-old students was found to be strongly related (effect size of about 0.8) to mathematical literacy. (Thomson, Cresswell & De Bortoli, 2004)

It is also important to note the distinction between *literacy in mathematics* and *mathematics literacy*. The former generally refers to students being able to access the mathematics in words and to make sense of the context and clarify what is required. This of course requires an understanding of the language used but also an ability to clarify the situation. The latter refers more to an aptitude or fluency and as assessed in the PISA programme. Research findings pertinent to these relationships follow.

At the upper primary level, Newman (1977) examined the errors made by students as they solved worded mathematics problems. There were seven categories of error which were related to the sequencing associated with problem solution: reading, comprehension, transformation, process skills, encoding, carelessness, and (lack of) motivation. She found that at least 35% of the errors made occurred before students were able even to attempt to apply mathematical skills or knowledge. These were language-based errors that occurred during the reading, comprehension, and transformation stages. Later research by Clements (1980) and Clarkson (1983) confirmed Newman's findings.

Doyle (2005) maintained that literacy, with respect to the ability to read a given text, was an essential part of the mathematical problem solving process:

the more students understand information, the greater chance they have of participating and developing mathematical skills (p.40).

Dawe and Mulligan (1997) attributed the fact that 6,000 out of 56,000 children failed to respond to a relatively straightforward item on a Basic Skills Test, largely to a failure of reading comprehension rather than mathematical inability. Mayer (2004) listed translating, integrating, planning, and executing as the cognitive processes used in mathematical problem solving. The first, translating, required linguistic and factual knowledge in how to convert the text into symbols. Unsuccessful problem solvers, Mayer (2004) suggested, tended to focus on the numbers when reading a problem and used keywords to decide what sort of operation to apply. Thus, for example, 'less than' would lead to a subtraction and the student would subtract the numbers in the order they were presented in the problem. A focus on the variable names and the relations between the variables with the consequent ability to apply the relational term less than to the correct numbers in the correct order would characterise the successful student.

Language factors specific to mathematics have been identified. DiGisi and Fleming (2005) described three types of vocabulary that students needed in order to be able to solve word problems: mathematics vocabulary, procedural vocabulary, and descriptive vocabulary.

In a recent study of Year 7 students' performance on the 2005 Victorian Achievement Improvement Monitor (AIM) test, Ong (2007) found that students from language backgrounds other than English (LBOTE students) performed better than English speaking background students on test items that had no words at all or which did not reflect linguistic or cultural bias.

Communication in the mathematics classroom is another dimension in which language and mathematics learning intersect. Contemporary understandings on how children learn mathematics well include the need to communicate mathematically. In mathematics curricula a balance between listening and speaking and other modes of communication are now advocated (e.g. small group discussion, writing about mathematics, oral presentations, and teachers' questioning techniques). Leder (1990) showed how allowing children to talk and listen to others talk about mathematics could be useful in monitoring progress, and identifying and analysing errors.



### **Recommendation 8:**

That the language and literacies of mathematics be explicitly taught by all teachers of mathematics in recognition that language can provide a formidable barrier to both the understanding of mathematics concepts and to providing students access to assessment items aimed at eliciting mathematical understandings.

## **2.5 Assessing student achievement**

Assessment is central to the teaching and learning process, for determining the standards students have reached and for improving what is taught and associated pedagogy. Much of the current research on assessment has focused on *assessment for learning*; classroom-based assessment which has at its focus improved learning outcomes for students (e.g. Black & Wiliam, 1998). At a policy level, however, increasing attention has been given to *assessment of learning* through statewide and national tests to determine the level of achievement by students at school, jurisdictional and national levels (MCEETYA, 2005). Many educationalists and a number of the submissions see a clear dichotomy between the two roles and argue that large scale-tests have no diagnostic role which results in the improvement of student outcomes (e.g. Shepard, 2000). Others, however, see the two roles as complementary and that any assessment, whether classroom-based or large scale test, can lead to improved teaching and hence higher student achievement; depending upon how data from the test are used (Masters, 2006). The quality of feedback, at the level of individual student, school or jurisdiction, is the determining factor.

### **Using test data for school improvement**

All states and territories assert that their tests, in addition to their monitoring role, served to provide diagnostic data at student and school levels. Nisbet (2004), for example, in the context of a review of the Queensland testing programme, described the government's goals as being to identify students' strengths and weaknesses, to provide data to inform planning and teaching, to provide results related to particular groups (e.g. boys/girls, non-English speaking background (NESB), Indigenous), and for the identification of teacher professional development needs. Other state and territory assessment authorities make similar claims.

Similar purposes are given for such assessments in other countries. Jennings, Price and Pankhurst (1999), for example, described purposes of the Numeracy Curriculum tests in England as including a formative role of recognising positive achievements, a diagnostic role of describing learning difficulties, a summative role of systematically recording overall achievement, and an evaluative role of reporting on the work of schools and Local Area Authorities. In the US, the *No Child Left Behind* (NCLB) Act requires states to test all students in Years 3 to 8 in reading and mathematics. Paulson (2007) argues that the effect has been to focus discussion on the needs of underperforming students whose needs had not previously been emphasised. Ruthven (1994) articulated a different supporting argument that externally set assessments can be a positive way of stimulating classroom reform. He cited, as an example, the positive effects that assessments that included practical work and investigation have had on school practices. It is noted that these commentators did not include reporting to parents on a student's progress as one of the benefits or purposes. Perhaps this aspect is taken for granted.

There are clearly potential benefits of such assessments and the challenge is to ensure that the assessments themselves, the system's use of the results, and the associated teacher learning, are designed to take advantage of these benefits. A number of researchers who see assessment for learning as consistent with current models of pedagogy based on constructivist principles, which emphasise understanding rather than rote learning, view formal tests as having undesirable curriculum and pedagogical consequences and do not lead to improved learning outcomes.

Nisbet (2004), described potential disadvantages of the use of the tests as the narrowing of the curriculum, teachers might teach to the test, some testing items might be un-aligned to the curriculum, and the potential misuse of results. The responses from the teachers surveyed showed that they did not have confidence in the numeracy assessment. Only one-quarter of the teachers felt that the results gave an accurate picture of the ability of the students. While most teachers showed the students how to complete the items, and most gave some practice questions, less than one-third reported that the assessments influenced what they teach, and even fewer reported that the assessments informed the way they teach and what they assess.

Two characteristics of the Queensland assessments that inhibited effective use by teachers were identified. One was that the results are returned to the school too late in the school year to influence planning and teaching for that year. The other characteristic was that teachers required expert assistance to use the results effectively. Doig (2006) similarly argued that teachers are overwhelmed by assessment results they are given and either treat the information superficially or do not use it at all. This aspect was explored by Williams and Ryan (2000) who studied the diagnostic potential of children's responses to national testing in England. They argued that many teachers do not use diagnostic methods and even seem unaware of their potential. Williams and Ryan argued that, as well as serving a system monitoring function, large scale assessments need to be designed to allow diagnosis of students' methods, and that the reporting of results should incorporate this information.

Similar concerns have been raised about the NCLB legislation in the US. Goldhaber (2002) noted that testing had the effect that the test content is emphasised at the expense of other, perhaps more important, content. Goldhaber (2002) also noted various school initiatives to improve test scores without necessarily improving teaching and learning, one effect of which can be to reduce the opportunities of higher achieving students. Menken (2006) argued that high-stakes tests have become the de facto curriculum policy in schools. Atkinson (2004) was critical of the effect of testing in stifling teacher initiative.

In contrast, other researchers argue that statewide tests can and do have an impact on pedagogy and student learning. For example, the results of a large survey (Cooney, 2007) of New South Wales schools in 2005 showed that parents and schools were using test data for diagnostic purposes at both student and school levels rather than for accountability. Parents saw the results as important in providing information about the strengths and weaknesses of their child's achievements and whether they were performing at the appropriate standard. How their child performed relative to other students in the state was regarded as less important than showing them the areas in which they could help their child improve. Principals and teachers also saw the test data as a valuable source of information for identifying students at risk, areas of their curriculum that need attention and teaching strategies that can be modified. In common with parents, principals and teachers saw performance against standards as more important than performance against other students or schools. Their open-ended comments showed that test results

were used extensively in whole-school planning with data from one test providing direction for the next planning period. The key to what is an increase in the use of test data by schools was the high quality of the student and school reports and the introduction of software that allowed schools to analyse their own data. These conclusions about the value of the statewide numeracy tests did not devalue the importance of school-based assessment. Respondents to the survey noted that statewide tests are only part of a school's assessment programme: they need to be seen in the broad assessment framework and in relation to current pedagogy.

The New South Wales survey did reveal differences between numeracy and literacy, with greater attention paid to the literacy results. Two reasons emerged from the analysis: firstly that literacy was perceived as more foundational, and there is research to support this view, and secondly that teachers were more comfortable in considering literacy across the curriculum rather than numeracy. For primary teachers it reflected their lack of confidence in their mathematical knowledge while for secondary teachers it reflected the view that numeracy was the responsibility of the Mathematics faculty (Cooney, 2007).

The findings from the New South Wales study are supported by research from other states and territories (e.g. ACER, 2003) and from many of the submissions. In the words of one respondent

There is also analysis of Basic Skills Test and Student Numeracy Assessment Programme results undertaken at regional and systemic levels. This allows for tracking of trend data, identification of schools which may require formal targeted intervention and focused provision of support from the system. Item analysis is undertaken by Numeracy Curriculum Officers to identify areas where students may be achieving well and areas which may need to be addressed at individual school level and across the system of schools. This information informs broader professional development in numeracy and mathematics [Catholic Education Commission, New South Wales, 22].

Although these researchers readily acknowledge the way that large-scale testing programmes can influence what schools and teachers do, and the undesirable consequences for curriculum and pedagogy that can result, they conclude that such programmes can be regarded as *assessment for learning* provided that the tests:

- mirror what is important and make rich ideas rather than items dominant
- ensure 'national' comparability data information is available (but avoid) league tables
- aim to enhance teaching and learning and ensure that teachers value the assessment as part of teaching
- assess what has been taught rather than teach what is to be assessed
- provide meaningful feedback *to all participants* (Hattie, Gavis & Brown, 2004).

In summary, the overall conclusion is that large-scale assessment programmes can result in improved student outcomes if they share the same qualities as good classroom assessment tasks. These qualities include a close relationship between what is taught and how it is taught, high quality items that allow the achievement of all students to be accurately determined against standards, and adequate and timely feedback to students and schools that supports their teaching and learning strategies. Timely and appropriate feedback to schools, so that schools could interrogate their own test results, was seen to be critical. Where teachers did feel not confident to carry out their own analyses there was less likelihood that the data from the statewide tests were used.

Williams and Ryan (2000) also argue that creating tests with a specific diagnostic role and providing information at the more detailed level of individual responses will inform teachers about the children's mathematical understanding, and that some patterns of student errors may give teachers some specific diagnostic insight. If this were the case we would argue that this type of test review could play a significant role in supporting and educating teachers, and in helping to lay the foundations for better practice (p.59). For example, Anghileri (2007) described some assessment items used in England that emphasise understanding rather than proficiency. One example was an item in which calculators can be used. The question was the equivalent of 'A pie costs \$2.40. How much would 3 pies cost?' The answer '7.2' that would appear in the calculator display is marked incorrect. Such an interpretation clearly gives an important signal to teachers about the meaningfulness of answers.

The AAMT submission asserted that national full-cohort tests are not sophisticated enough to provide the detailed data required to systematically monitor and provide feedback for intervention and improvement programmes in schools. They recommended that a national assessment programme based on light sampling methodologies should be established to ensure that quality, targeted evidence is available in the future which will address a wider range of mathematics/numeracy outcomes. While their assertion can, and has been, challenged by other researchers, their recommendation has some merit; not to replace the full-cohort tests but to supplement the tests [AAMT, 31].

In summary, large-scale tests can serve useful purposes, but they need to be well constructed, have both instructional and monitoring purposes and be supported through teacher professional development and timely and appropriate feedback to schools and jurisdictions. There also needs to be a recognition that well-structured classroom assessment is by far the most important and useful form to the classroom teacher.

## Classroom assessment

Current research shows clearly that a requirement of high quality classroom-based assessment is that it is an integral part of the teaching and learning cycle, and that appropriate and timely feedback is provided to students. This is evident in relatively recent work in Queensland with their *rich assessment tasks*; in the research conducted by the Assessment Reform Group in the UK, and the Curriculum Corporation in Australia to name but a few. To assist teachers to access the complex weave of classroom activities involving pedagogic style, student-teacher interaction, self-reflection, motivation and a variety of assessment processes the Assessment Reform Group has enunciated ten principles of assessment for learning:

- part of effective planning
- focuses on how students learn
- is central to classroom practice
- is a key professional skill
- is sensitive and constructive
- fosters motivation
- promotes understanding of goals and criteria
- helps learners to improve
- develops the capacity for self-assessment
- recognises all educational achievement (Gardner, 2005, p.5).

These points are primarily about pedagogy – the following four questions can also be used by the classroom teacher to inform the process of assessment:

- What do you want the students to learn?
- Why does the learning matter?
- What are you going to get the students to do (or to produce)?
- How well do you expect them to do it? (New South Wales Department of Education and Training, 2004, p.10)

These questions highlight the primacy of curriculum, the importance of a focus on the learner and the use to which information from the test will be used. In an effective assessment programme, desired learning outcomes are identified, the assessment strategy chosen is consistent with pedagogy and appropriate to the outcomes being assessed, student achievement is judged against the desired learning outcomes, and timely feedback is provided to students and teachers alike. The focus is on students and their improvement.

The role of assessment is important in mathematics motivation and attainment. Baker, Gersten and Lee's (2002) meta-analysis suggests that the use of attainment data and other forms of performance and feedback is highly effective in lifting the achievement of low attaining mathematics students. Although one of the smaller meta-analysis studies, examining just 15 studies, the quality of the evidence was high and the effect sizes were amongst the strongest seen. Interestingly, there is evidence that the feedback is equally effective when used by parents and students, as it is when used by teachers. Several reviews have found that assessment practices can have an impact on students' attitudes and achievement (e.g. Crooks, 1988; Natriello, 1987) in both positive and negative ways depending on the nature of the feedback. There is evidence that teacher subject matter knowledge and pedagogical content knowledge are both required for teacher assessment and feedback to be effective (Anthony & Walshaw, in press). Further reviews of research (e.g. Bangert-Drowns, Kulik, & Kulik, 1991; Black & Wiliam, 1998; Kluger & DeNisi, 1996) have explained in what circumstances assessment helps and when it hinders students' learning. Black (2002) reported findings from teachers to show how their roles as teachers and the roles of their students as learners had been transformed. The research showed that there were four components of change:

- Teachers asked questions in class, giving pupils time to think about a question and expecting everyone to respond.
- Teachers marked homework, concentrating on giving comments on which pupils were expected to take action to improve the work.
- Pupils assessed one another, including marking each others' work.
- Pupils were involved in constructing their own tests.

Most recently, Wiliam and Thompson (2007) synthesised research on how assessment can support numeracy learning. They suggested that formative assessment produces greater benefits for student learning than class-size reduction or increases in teacher content knowledge and that the gains arising from short- and medium-cycle formative assessment was approximately double that found in other classrooms. They proposed a typology of three types of assessment: long-cycle (4 weeks to a year), medium-cycle (1 to 4 weeks), and short-cycle (within and between lessons). Short-cycle assessments were further divided into day-by-day and minute-by-minute categories. Wiliam and Thompson argued that only short- and medium-cycle assessments improve student achievement.

Three processes were found to be central in providing a comprehensive framework for formative assessment: establishing where learners are in their learning; establishing where they are going; and establishing how to get there. By considering the role of the teacher, the student and the student's peers separately, Wiliam and Thompson (2007) built up five 'key strategies' as shown in Figure 2.

**Figure 2 Aspects of assessment for learning. (Wiliam & Thompson, 2007)**

	Where the learner is going	Where the learner is right now	How to get there
Teacher	Clarifying and sharing learning intentions and criteria for success	Engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of learning	Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	Activating students as the owners of their own learning	

Classroom-based assessment can enable teachers to pay attention to the knowledge and beliefs that learners bring to a task, to use this information as the starting point for new instruction, and monitor students' changing conceptions as instruction proceeds. Good practice in assessment demands that teachers develop a learning culture by having assessment based on theories of learning (Shepard, 2000). To enable the tracking of student achievement over time, well-constructed learning continua (quantitative maps) that are qualitatively described are desirable according to Rowe [Rowe, 47]. The use of such maps provides deeper understandings of learning progress than can be obtained from cross-sectional snapshots that merely assess the achievements of students at different point in time. They also stand in contrast to the more traditional curriculum-based approaches that impose a list of outcomes that students are expected to learn, and then test to see whether these outcomes have been achieved.

The Middle Phase of Learning Cluster project was initiated in Queensland to provide access to high-quality professional development in literacy, numeracy and assessment for middle years' teachers in seven clusters. Drawing on contemporary research, the project design acknowledged the changing nature of literacy and numeracy in the context of new technologies; the necessity to explore the literacy demands that students encounter in different learning contexts and disciplines; and the need to incorporate these explicitly in assessment. The final report for this project [Education Queensland, 39] encompasses a number of recommendations that teachers can use in their assessment practices:

- Front-ending assessment, that is, linking assessment to learning from the start of the learning journey, was considerably effective in improving student understanding, engagement and learning outcomes.
- Identification and explicit teaching of the curricular knowledges as well as curriculum literacies, and/or numeracy demands, of the task facilitated greater understanding and achievement of the task requirements.
- Explicitly linking the task and the criteria and standards and criteria helped students to be clear about what was expected of them. This enables students to work independently and monitor their own progress.
- Developing and using explicit standards and criteria which enable teachers to make more accurate and reliable judgements about student achievements.



- Foregrounding, explicitly teaching and scaffolding student learning of the curriculum knowledges and literacies, helps students to become far more confident when dealing with intellectually challenging experiences.
- Developing a deeper understanding by teachers of the alignment of curriculum, pedagogy and assessment provides a reliable basis for communicating the task requirements to students and parents.

Several submissions [e.g. Catholic Education South Australia, 4; AAMT, 31; Curriculum Council Western Australia, 32; DET New South Wales, 35; Tasmanian Department of Education, 43; Catholic Education Commission, Victoria, 50] demonstrated how these recommendations were being implemented, including the use of 'rich' assessment tasks that were multi-level in design and contextualised to allow students to apply their mathematical knowledge and skills and open-ended tasks that allow all students to access aspects of the tasks and to demonstrate the use of a range of solution strategies, techniques and tools.

In their report of the TIMSS Video Study of 638 Year 8 lessons from seven participating countries Hollingworth, Lokan, & McCrae (2003) supported the need for complex teaching, learning and assessment, noting that, 'Australian students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking' (p.xxi). They further commented, however, that 'there is an over-emphasis in Australian Year 8 mathematics, as in some of the other countries, on 'correct' use of the 'correct' procedure to obtain 'the' correct answer. Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on are rare.' (p.xxi) They reported 'a syndrome of shallow teaching, where students are asked to follow procedures without reasons' (p.xxi).

The report concluded that, despite recent developments in some Australian mathematics curricula which have resulted in a greater emphasis on thinking mathematically, many teachers report considerable pressure to focus on superficial learning rather than a more in-depth knowledge of mathematical concepts. The aim for many teachers is still 'getting through the course'. Their analysis of the 87 Australian mathematics classrooms videoed showed that a large proportion (75%) of problems were low in procedural complexity. This was the highest of any country in the study. Just over a quarter of problems used real-life connections (compared to 42% in The Netherlands) and less than 10% of problems had more than one solution (Hollingworth et. al., 2003).

It is difficult to reconcile the contrasting data – it is possible that practice has changed since the TIMSS study years, or it is possible that the submissions are reflecting what *should be* rather than *what is*. Evidence from one submission gives some support for the second alternative – *teaching practice has not caught up with curriculum change*.

The need to improve assessment practice in schools through professional development and the provision of exemplars is well documented. The *Balanced Assessment for the Mathematics Curriculum* project in the US aims to change the focus of teaching so that all aspects of mathematics are valued, to provide exemplars of best practice in assessment, and to link ongoing assessment with teaching strategies (The Concord Consortium, 2007). Several submissions to the review [Office of the Board of Studies, New South Wales, 34; New South Wales Department of Education and Training, 35; Department of Education Victoria and Victorian Curriculum and Assessment Authority, 48] provided evidence of the development of similar websites in Australian states and territories. Such initiatives have obvious merit and should be encouraged.

In summary, there is no argument with the proposition that assessment has a powerful effect on what occurs in schools. It follows, therefore, that if all aspects of mathematics and numeracy are to be valued, that assessment practices, both classroom-based assessment and formal testing programmes, must give attention to all. It is noted that the evidence from research and the submissions to the review, that improvements are required in assessment practice, at both school and jurisdictional levels.

## School entry assessments

Data both locally (e.g. Clarke et al., 2002) and internationally (e.g. Aubrey, 1994) indicate that there are wide differences in the numeracy knowledge of students on school entry and that, without specific intervention, teachers do not identify and make use of knowledge about differences. There is a range of school entry assessments in Australia and New Zealand, most of which use a form of clinical interview. One exception is the I can do maths assessment for which a pencil and paper format is used.

The *School Entry Assessment* (Ministry of Education, 2003) was introduced into New Zealand primary schools in 1997 to determine the 'nature and extent of certain knowledge and skills shown by new entrant children when they begin school' (p.3). It has three components: literacy; numeracy; and oral language. For the numeracy component a game format task termed *Checkout* is used. Individual students' results are scored and can be used to 'gauge new entrants' skills in, and understanding of, selected aspects of early ... numeracy' (p.6). The overall assessment is used in close to 60% of NZ schools; although less than one-third send results to the Ministry of Education. In a survey (Ministry of Education, 2003), the majority of teachers suggested that the assessment overall needed to be updated, and that *Checkout* should be adapted to reflect the Numeracy curriculum.

The *Early Numeracy Interview* was developed as part of the Victorian Early Numeracy Research Project (Clarke et al., 2002) and was designed as a research tool to collect data over the first three years of school. To address the diversity of needs on school entry, a particular set of questions was developed. Clarke et al. reported that:

the interview enabled a very clear picture of the mathematical knowledge and understandings that young children bring to school, and the development of these aspects during the first year of school. Most Prep children arrive with considerable skills and understandings in areas that have been traditional content for this grade level. As acknowledged by many trial school teachers, this means that expectations could be raised considerably in terms of what can be achieved in the first year (p.25).

This interview has now been adapted by the Department of Education (Victoria) as the *Mathematics on-line interview*, and teachers can input results to a central database that provides both central data and allows possibilities for comparisons. The evidence from the Victorian experience is that the early assessment of students is a sufficiently powerful information tool that schools and teachers are willing to overcome the organisational challenges of one-on-one interviews.

There is a similar approach in New South Wales, *Count Me In Too*, which is described by the New South Wales Department of Education and Training (2007) as an:

'innovative numeracy project operating across New South Wales Department of Education and Training primary schools. It is designed to assist teachers to broaden their knowledge of how children learn mathematics by focusing on the strategies students use to solve arithmetic tasks. The project aims to improve the educational outcomes in mathematics for all students through



professional development of teachers. It achieves this by increasing teachers' understanding of how children develop increasingly sophisticated ways of solving arithmetical problems. The research-based learning framework used in the project provides direction for teaching and learning' (New South Wales Department of Education and Training, 2007).

The project also has an individually administered interview, the Schedule for Early Number Assessment, the results from which are used to inform planning at classroom level and individual level using the *Count Me In Too* learning framework.

Two other Australian developed assessments that have been used successfully with school entrants were described by Thomson, Rowe, Underwood and Peck (2005): *Who am I?* and *I can do maths*.

In summary, the evidence shows that school entry assessments have potential for informing the teaching and learning of numeracy, and that appropriately constructed school entry assessments, along with adequate school and system support for teachers to administer the assessments, and associated teacher professional development, would assist teachers in supporting the subsequent learning of all students.

#### **Recommendation 4:**

That a balanced view be taken of the relative contributions to effective student learning of systemic assessment programmes and high quality classroom assessment in the allocation of resources to develop and support each.

#### **Recommendation 5:**

That the necessary resources be directed to support teachers to use diagnostic tools including interviews to understand and monitor their individual students' developing strategies and particular learning needs. These diagnostic tools should not be restricted to school-entry assessments.

#### **Recommendation 7:**

That systemic assessment programmes be extended to include sampling of students to provide more in depth information about common conceptions and misconceptions, and areas of difficulty for students, with the purpose of providing (a) a research base to inform ongoing curriculum development and pedagogy and (b) improved diagnostic tasks for individual teacher use with students in their classrooms.

## 2.6 The pedagogies of adult numeracy

In this section the terms ‘mathematics’ and ‘numeracy’ are used interchangeably to reflect the terminologies of the various authors in the field. Internationally, recent years have seen the growth of a considerable body of research into how adults learn mathematics/numeracy in formal learning situations as well as informally elsewhere, and what might be considered as optimal pedagogical approaches to support this learning. One critical difference between teaching adults returning to study and school-age children continuing their studies is that adults have formed a reservoir of considerable life experience and are generally strongly motivated to make meaning within these learning situations. Another is that many adults returning to study mathematics/numeracy have had prior experiences of formal education which may have been less than successful, and so are likely to bring strong affective loadings in the form of beliefs, attitudes, and emotions concerning both the discipline of mathematics and their own identities as learners in this particular context – even though they generally regard themselves as competent adults in other spheres of life. The term *mathematics anxiety* is commonly used to refer to the negative connotations of this phenomenon but, as with the definition of numeracy, there is no universal agreement on a single concept and international research continues to be undertaken into the affective domain – from psychological as well as socio-cultural perspectives. Clearly, it is important that teachers of adults (and children also) take into account both cognitive and affective aspects in their pedagogical practice.

Given the uniqueness of each individual’s life trajectory, instructional groups are likely to be diverse – in terms of social, cultural, economic and educational background, particularly where there are newly arrived immigrants for whom schooling may have been minimal, on the one hand, or extended to university studies, on the other. In workplace education especially, there may be power relationships at play when people of different status are combined in the one mathematics/numeracy class. Teachers have a responsibility to find out what relevant mathematics and other knowledge and skills their learners bring to the topic at hand, but it is also essential to maintain good relationships with adult learners and promote a respectful, harmonious atmosphere even when there are intellectual tensions or conflicts as part of the learning process.

In any educational field, it is common for adult educators to learn things they did not know from their students. Thus, an obvious requirement for mathematics/numeracy teachers is to listen actively to their students and to promote conversations – some researchers and teachers have developed and theorised these ideas as *dialogical learning*, based on the work of Freire (1998).

For many years there has been a tendency to associate adult numeracy education at the ‘entry’ or ‘basic’ levels with the embedding and reinforcing of particular skills so that they become routine – usually this is in number and measurement, but it can also apply, for example, at the undergraduate level with calculus skills. It is important to enable adults at all cognitive levels to have the opportunity to move to higher levels, to be able to solve problems in a changing workplace. Pedagogies for adult numeracy need to be flexible, situated in contexts which are meaningful to the particular learners, take account of their previous knowledge and experience, challenging them to develop higher levels of understanding, and ultimately to communicate with others in authentic situations.

## 2.7 Computational and information and communication technologies

The use of information technology in mathematics has been the focus of substantial study and there is evidence, of notably diverse quality, which supports the use of ICT to supplement and support both mathematics motivation and mathematics attainment. The incorporation of ICT in the teaching and learning of mathematics in schools is, however, not simply a reflection of emerging tools in the new millennium. As the Victorian Middle Years Numeracy Project report indicated, ‘further gains in numeracy performance could be achieved if ... technology was explored more specifically in relation to numeracy teaching and learning’ (Victorian Department of Education Employment and Training et al., 2001, p.84) Any consideration of the links between ICT use in school mathematics lessons and learning outcomes has to be made in the context that the way ICT has been incorporated into lessons has changed over the years in response to educational change. White (2005) summarised this phenomenon by asserting that mathematics educators:

used ICT initially for drill and practice, based on behaviourist theories and outcomes concentrating upon mastery of skills. Then the tool, tutor and tutee models ... became popular as these promoted higher order thinking and more student centred learning. This trend was supported by psychological theories about information processing ..., cooperative learning and metacognition (p.231).

### Computational calculators in mathematics lessons

The different states and territories in Australia have the same policy relating to computational calculator use in the respective examinations, both in terms of condition of use and permitted models (Barrington & Brown, 2005). The use of calculators for all school levels in Australia was endorsed by all states and territories in 1986 (Curriculum Development Centre & Australian Association of Mathematics Teachers, 1986), and later in the *National Statement on Mathematics for Australian Schools* in 1991 (Australian Education Council & Curriculum Corporation, 1991). Data collected from participating Australian schools in 1999–2000 for the TIMSS 1999 Video Study, however, indicated that computational calculators were only used in 56% of Year 8 lessons in Australia. There was also no correlation found between the extent to which computational calculators were used in Year 8 lessons of participating countries and the countries’ respective performances in the comparative study (Hollingsworth, Lokan, & McCrae, 2003). This conclusion was also reflected in primary mathematics lessons through the British *Leverhulme Numeracy Research Programme* (Brown, 2000).

The use of simple calculators, however, has accounted for ‘a major change in primary schools in the past twenty years’ (Groves, Mousley, & Forgasz, 2006, p.94). The 80 teachers and 1000 students who took part in the 1992–1994 *Calculators in Primary Mathematics* project (see Groves, 1995) showed how facilitating children access to hand-held computational calculators in the early years led to significant and profound contribution to understanding, skill and performance. These achievements included success at mental computational tasks.

Despite this and other projects (such as the *Calculator-Aware Number* project in England in the late 1980s) which point to pedagogical benefits of harnessing computational calculators in the primary mathematics classroom, this resource is not used in schools as often as desired by such researchers. In many primary schools, such calculators ‘are used only for checking already-completed work or for special calculator activities’ (Groves, Mousley, & Forgasz, 2006, p.94). Teacher beliefs in this regard remain an issue (Sparrow & Swan, 1997), or, more specifically, there was an observed discrepancy between teachers’ espoused beliefs and their actual use of the tool in their respective classrooms (Stacey & Groves, 1994).

## Computers in mathematics lessons

Despite the recommendations of the *National Statement on Mathematics for Australian Schools* (Australian Education Council, & Curriculum Corporation, 1991), the impact on student outcomes of incorporating computers into classrooms might not be conclusive, and is also not easily measured. Forgasz, (2006a) proposed that the extent to which computer use enhances attainment might be conditional upon student characteristics and/or software types. The use of computers in mathematics classrooms over 1999–2000 in Australia (and across many other countries) has been relatively low, as indicated by the TIMSS 1999 Video Study (Hollingsworth, Lokan, & McCrae, 2003). A three-year study conducted later in middle schools in Victoria and Queensland pointed to a continuation of this trend: ‘given the research that has been conducted with these tools in terms of their capacity to bring about rich mathematical understandings, what struck us was their minimal uptake in all schools in this study, regardless of the demographics of the schools’ (Zevenbergen & Lerman, 2006, p.596).

Compounded with low-usage rates is the manner in which computers are used in the classroom. Even with increased use of such technology in schools, Ertmer’s (2005) study found that teachers are ‘using technology for numerous low-level tasks (word processing, Internet research) [whereas] ... higher level uses are still very much in the minority’ (p.399).

Zevenbergen and Lerman (2006) drew on the Victorian and Queensland data to suggest that among the factors contributing to this phenomenon were confidence and skills level with ICT, thus suggesting the possibility of teacher in-service professional development being a potential solution. In fact, the design of teacher professional development programmes might benefit from recent research with secondary school teachers (Forgasz, 2006a) which indicated that factors which are promoting computer use for some teachers are also the inhibiting factors for the same purpose for other teachers! More importantly, ‘the most prevalent encouraging and inhibiting factors that emerged were strikingly similar to those reported in earlier research studies on computer use for education generally ... and for computer use for mathematics teaching more specifically (Forgasz, 2006a, p.89). Encouraging factors included availability of good quality, motivating and fun software, availability of computers and computer laboratories, and teachers’ own confidence and skills. Inhibiting factors included poor software, lack of access to computers and poor quality equipment, and a need for professional development.

## Graphic calculators and Computer Algebra Systems (CAS) in mathematics lessons

In many educational systems around the world, graphing calculators and CAS have progressively been introduced into secondary school mathematics classrooms over the last decade or so. In Victoria, for example, in the first two years of secondary schooling students are typically expected to:

use technology such as graphic calculators, spreadsheets, dynamic geometry software and computer algebra systems for a range of mathematical purposes including numerical computation, graphing, investigation of patterns and relations for algebraic expressions, and the production of geometric drawings (Victorian Curriculum and Assessment Authority, 2005, p.31).

In the next two grade levels of secondary schooling:

students use technology (for example, geometry software, graphics calculators, spreadsheets and computer algebra systems) to develop mathematical ideas and solve problems (Victorian Curriculum and Assessment Authority, 2005, p.34).

The 'rolling out' of a new mathematics subject in Victoria, namely *Mathematics Methods (CAS)*, to all schools in 2006 followed three years (2000-2002) of trialling in three schools using three different models of CAS. The trialling was part of a research project funded federally under the Australian Research Council (ARC) Strategic Partnerships with Industry–Research and Training scheme. By providing participating students and their teachers with CAS, so that all had access to this form of technology in all facets of classroom teaching, the focus of this project was to document any change to learning, teaching and assessing. Generally positive findings supporting the introduction of CAS in secondary school classrooms were reported (e.g. Stacey, 2001), and this was also the case for undergraduate mathematics classes (see Pierce & Stacey, 2001). The project also harnessed such valuable knowledge to devise and trial the new subject, *Mathematics Methods (CAS)*.

In 2006, New Zealand's Ministry of Education carried out an evaluation of its 2005–2006 CAS Pilot Project (Neill & Maguire, 2006). All the teachers from the Project reported that the CAS calculators were supporting and enhancing mathematics pedagogy of a more exploratory, discovery-based approach. For the students, they reported a general interest and confidence in using CAS in their mathematics learning. This did not mean that students felt that their understanding in mathematics increased, however, a significant minority (mainly the more able students who had benefited from the more 'traditional' lessons) expressed their concern that their understanding had decreased. In terms of student outcomes, schools participating in New Zealand's CAS Pilot Project reported that their CAS students performed as well as non-CAS students on algorithmic questions.

Ellington's (2006) review of 42 studies evaluating the impact of the use of calculators found that use of graphic calculators during assessment helped lift both the conceptual and procedural performance of students. The impact of graphic calculators, while beneficial, is modest in relation to other ICT interventions. There is substantial evidence to support the use of Computer Assisted Instruction (CAI) in lifting the attainment of 'at risk' students and students with special educational needs. Some moderate to strong effect sizes have been demonstrated for this group (Kroesbergen & Van Luit, 2003) for this group with small to moderate effect sizes for broader populations (Slavin & Lake, 2007; Slavin et al., 2007) and for lifting secondary students' motivation in mathematics (Kyriacou et al., 2006).

To put these benefits into perspective, several meta-analytic reports compared CAI with other forms of intervention. All showed that the effect of instructional reform was substantially stronger than the effect of CAI (Haas, 2005, Kroesbergen & Van Luit, 2003, Slavin & Lake, 2007; Slavin et al., 2007) CAI has a stronger effect on low attainers, for example, but these were not as strong as those for some programmes that did not involve any ICT or CAI at all (Kroesbergen & Van Luit, 2003). Despite this, a consensus view emerges that CAI and ICT provide valuable supplementary benefits that may complement instructional reform.

## **Delivering mathematics lessons through ICT**

The emergence and growth of virtual schools in Australia and elsewhere represent a powerful and pervasive attempt in integrating ICT to the teaching of various school subjects, including mathematics. The virtual learning application of ICT in (mathematics) education is also related to the facilitation of distance learning for students located in the remote outback regions of Australia and in other nations. In virtual schools, 'students spend part or all of their time working 'off-campus', for example, from home using

an online computer' (Russell & Finger, 2003, p.3). There does not seem to be any research conducted to assess the extent to which mathematics is taught or learnt particularly effectively through virtual schools, and some reported negative effects of this mode of lesson delivery on students may negate any gains<sup>5</sup>.

## The role of affect and technology

Regardless of the real extent to which technology use enhances the teaching and learning of mathematics in schools, it is worth remembering that this potential hinges on the related technology being used in the classroom in the first place. Barriers to this may be institutional, where individual schools are not able to secure the necessary funds to acquire or to maintain the necessary hardware and/or software. On the other hand, teachers provide another layer of potential barrier to technology use and integration in mathematics lessons. Factors related to this include the provision of, and teacher access to relevant teacher professional development (White, 2005), teacher beliefs about the nature of mathematics (Yang, Butler, Cnop, Isoda, Lee, Stacey, & Wong, 2003), about mathematics pedagogy (Baturu, Cooper, Kidman, & McRobbie, 2000; Forgasz, 2006b), and teacher beliefs about student gender (Forgasz, 2006b). In relation to the last point, Forgasz's study with 111 secondary school mathematics teachers indicated that amongst those who held the view that boys and girls work with computers differently, there was a general sense that girls displayed less confidence, less competency, and less interest in using computers when compared to their male peers. This suggests problems for integrating ICT into mathematics lessons, since it might exacerbate issues some girls have with mathematics.

Indeed, teachers' affective responses to the use or integration of ICT in mathematics lessons can (and do) play a key role in determining the extent to which related policies are successfully executed in the classroom. For example, in the late 1990s when the Singapore Ministry of Education recommended that some 30 per cent of curriculum time should feature ICT use, 'some mathematics teachers try to satisfy this [policy requirement] by using PowerPoint as a presentation tool, which is usually not effective to teach pupils how to solve problems' (Yang et al, 2003, p.61), which at the time was the central focus of mathematics education in Singapore.

## 2.8 Ability grouping

There is widespread adoption in Australia of ability grouping ('streaming' or 'setting'), which refers to the practice of grouping the high achieving students together, and the lower achieving students together in the same and/or different classrooms. There is substantial international research that suggests the practice does not enhance the learning of students, and indeed, may hinder learning particularly for students in the lower streams (Boaler, 1997b; Boaler, Wiliam, & Brown, 2000; Gamoran, 1992; Slavin, 1990, Kyriacou et al, 2006)). In studying the practice of ability grouping in Australian schooling, Zevenbergen (2003; 2005) showed that students in the lower streams identified issues around teacher quality, pacing of content, examination of content covered (or not covered) that contributed towards students' perceptions of themselves as learners of school mathematics. Boaler (1997a) found that even in the upper streams, there is not full support for learning in these streams.

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5 For example, Kraut et al. (1998) warned of virtual school students experiencing increased feelings of loneliness, depression and anxiety, as well as poorer social relationships. Salomon (2000) argued that there is some question of students possessing the necessary self-discipline or motivation to learn via online computers. Students in Hosking's (2002) study reported a loss of opportunities to engage in immediate personal interaction with teachers, although the question remains open as to whether students might get used to such a form of mathematics learning over time to the extent that content is learnt just as effectively as in face-to-face learning situations.



Chen and Goldring (1994) suggest that there is wide acceptance of streaming by teachers. One reason why there is this support can be connected to the view that mathematics is hierarchical in structure (Ruthven, 1987). Coupled with the contemporary emphases in education where students' progress is mapped against levels, there is a congruency between teachers' beliefs about curriculum organisation, student learning and assessment. The hierarchy of learning approach leads to a belief that appropriate learning activities and scaffolding can be developed to move the students on to greater levels of understanding and competence (Slavin, 1990).

This is also connected to perspectives on ability. Underpinning the justification for streaming is a teacher belief in the notion of an innate ability whereby the students' abilities in mathematics is the major reason for the performance in mathematics (Lorenz, 1982). Accordingly, having students clustered around their 'natural abilities' is seen to allow teachers to construct learning activities that match the perceived ability of the students.

In their comprehensive review of ability grouping literature, Ireson and Hallam (1999) claim that there is no conclusive evidence to support or dispute the value of streaming in increasing academic achievement. In contrast, a large-scale study of American youth found that ability grouping helped the advanced students and harmed those in the low streams, and overall, had a negligible effect (Hoffer, 1992). Similarly, in her study of middle school mathematics classes, Burkes (1994) noted that students from the high-ability classes were more likely to view mathematics positively, engage in appropriate behaviour and undertake homework than their peers in middle or lower streams.

William and Bartholomew (2004) reported on 955 students followed from Year 8 to Year 11 in six London secondary schools. The project collected data through questionnaires, interviews of 100 students, 150 lesson observations and performance end of Year 9 and 11 on National Curriculum tests. They noted that 'the data reported here provide further evidence that ability grouping does not raise average levels of achievement, and, if anything, tends to depress achievement slightly, which is entirely consistent with results from studies conducted in the 1960's and 1970's in the UK, and with more recent studies conducted in the US. More importantly, this study replicates a key finding from earlier studies, that while ability grouping in mathematics has little overall effect on achievement, it does produce gains in attainment for higher achieving students at the expense of lower attaining students' (p.290). They also commented that 'in this context it is worth noting that every country that outperforms England in mathematics makes less use of ability grouping' (p.291).

The work of Slavin (1990) is among the best known in summarizing the research on ability grouping. He provides 'a comprehensive review of all research published in English that has evaluated the effects of ability grouping on students achievement in secondary schools' ... Overall achievement effects were found to be essentially zero at all grade levels. Results were similar for all subjects except social studies for which there was a trend favouring heterogeneous placement. [However] tracking generally has a positive effect for high achievers and negative for low achievers (p.471).

In her study of the effects of ability grouping, Davenport (1993) noted that 'the report identified three areas in which strong inequities in mathematics instruction were found: (i) access to strong mathematics programmes; (ii) access to well-qualified mathematics teachers; and (iii) access to classroom opportunities' (p.2). 'The study also found that schools often place their least qualified mathematics teachers in low-ability classes and their most-qualified teachers in their high ability classes, particularly at the secondary level' (p.2). 'With regard to mathematics instruction, a case study of one particular classroom showed that low-ability students received less teacher time and were asked a fewer number of process-oriented questions'. (p.3)

Zevenbergen (2005) argued that 'the objective practices of school mathematics create an environment through which students internalize the practices to develop a sense of self, a habitus. This habitus potentially is more or less empowering, depending on the experiences within the streamed setting' (p.607). 'Within the Australian context, grouping by ability is not enacted through any formal policies at state or federal levels, but remains the domain of individual schools. As such, schools are able to enact their own practices in terms of how classes are organized. In spite of this autonomy, the practice of ability grouping is commonplace in mathematics classrooms. This can be by way of year-level groupings or within-class groupings, depending on the school' (pp.607-608). Following interviews with 96 students across 6 schools at Years 9 and 10, Zevenbergen claimed that 'the practice of ability grouping helps to produce the status quo, and can be detrimental to goals of social justice' and that 'when the practice is enacted in mathematics classrooms it can create a learning environment that becomes internalized as a mathematical habitus' (p.608). 'The responses [of student interviews] confirmed the general understanding that students in the high streams reported positive experiences; were exposed to significant mathematical content; performed better in tests; and considered the discipline as relevant. The converse was true for students in lower streams. The trends held, regardless of the school, the year level, or gender' (p.612).

When Second International Mathematics Study data was analysed by Boaler, William and Brown (2000), it was suggested that 'the two factors that are most strongly associated with growth in student achievement in mathematics (indeed the only two factors that are consistently associated with successful national educational systems) are opportunity to learn (i.e.. the proportion of students who have been taught the material contained in the tests) and the degree of curricular homogeneity (i.e. the extent to which students are taught in mixed-ability, rather than settled, groups' (p.646).

In a meta-analysis involving 165 studies across a range of subject areas, Lou, Abrami, Spence, Poulsen, Chambers and d'Apollonia (1996) explored the impact of within class grouping. Students working in small groups achieved significantly more than students not learning in small groups, with the grouping effect (both cognitive and affective) being greater in larger classes. In summary, low ability and average ability students learned significantly more in mixed ability groups, while for high ability students, group ability composition made no difference. Interestingly, there were no significant differences between ability grouping and mixed ability grouping in mathematics, compared to significant differences in reading in favour of homogeneous groups.

In its submission to this review, the peak research body in mathematics education in Australasia, the Mathematics Education Research Group of Australasia (MERGA), while endorsing the desirability of upper secondary students to choose levels of study that are appropriate for their needs, interests and levels of attainment, does not support ability grouping in the primary and middle years of schooling. They note that ability grouping and setting are not usually used in primary and lower secondary classrooms in countries 'whose TIMSS and PISA results we would wish to emulate' [MERGA, 38, p.4].

### **Recommendation 9:**

That the use of ability grouping across classes in primary and junior secondary schooling be discouraged given the evidence that it contributes to negative learning and attitudinal outcomes for less well achieving students and yields little positive benefit for others, thus risking our human capital goals.



## 2.9 Fostering positive student motivation

Motivation is often perceived to be an important affective factor in facilitating cognitive development amongst mathematics students. The PISA 2003 study developed two indices to assess the relationship between students' intrinsic and extrinsic motivations on the one hand, and their mathematics performance on the other. The *interest and enjoyment in mathematics* index focuses on students' intrinsic motivation, and amongst the four countries whose performances were significantly higher than Australia's, students in three of them (the Netherlands, Finland, and Korea, but not Hong Kong) 'performed at a high level in mathematics but expressed less *interest and enjoyment in mathematics* than students in other OECD countries ... In Australia, there was a relatively weak positive association between the *interest and enjoyment in mathematics* index and mathematics performance' (Thomson, Cresswell & de Bortoli, 2004, p.184).

Similar results can be said of the parallel investigation into the relationship between extrinsic motivation and mathematics performance. Students from three of the four higher performing countries (compared to Australia) (i.e. the Netherlands, Korea and Hong Kong, but not Finland) scored means that were below the OECD average. Further, in Australia, the positive relationship between extrinsic motivation and mathematical literacy performance was similarly weak (Thomson, Cresswell & de Bortoli, 2004).

These positive relationships between different forms of student motivation and mathematics performance in Australia have also been evident in local research projects. The success of projects such as the Early Numeracy Research Project (ENRP) in Victoria has been associated with improved student affective states towards mathematics and the learning of mathematics. In the ENRP, students' development beyond their respective progressions along the growth points was documented. Amongst the five most common themes that were categorised from the student responses were three which related to increased levels of motivation: 'children enjoy maths more, look forward to maths time, and expect to be challenged; the development of a 'give it a go' mentality ... with greater overall persistence; ... all children are experiencing a level of success' (Bobis et al., 2005).

The extent to which motivation leads to better mathematics performance, however, has also been questioned by such researchers such as Stevens, Olivárez Jr and Hamman (2006). Their findings led the researchers to propose that mathematics self-efficacy, the extent to which students believe that they can solve mathematics problems, whatever the circumstances, is the strongest predictor of mathematics performance, stronger than general mental ability, and also stronger than intrinsic motivation.

Unlike beliefs and values, affective variables such as motivation, self-concept and attitudes are relatively unstable (McLeod, 1992). This is good news for any attempt at modifying students' motivation and self-concept in learning mathematics. Craven, Marsh and Debus (1991), for example, reported the short time needed for improvement in students' self-concept when their teachers' feedback to students was based on positive ability and on performance.

Student motivation to learn mathematics can come from different sources, the teacher being one of them. In the US, Middleton and Spanias (1999) found that 'student motivation in mathematics is highly influenced by teachers' instructional practices. If appropriate practices are *consistent over a long period of time*, children can and do learn to enjoy and value mathematics' (p.75). In Australia, Burnett (1999) studied the 'self talk' of 269 Australian children, in Years 3 to 7, in reaction to the frequency of their teachers' positive statements, and found these to be more influential than parents' or peers' comments. Positive statements made by teachers were more influential than negative statements.

Like their colleagues overseas, Australian teacher participants in the TIMSS 1999 Video Study were found to report goals in their lessons which would be categorised as content goals and process goals, while perspective goals were identified by very few of these teachers (Hollingsworth, Lokan, & McCrae, 2003). This phenomenon has implications for the fostering of positive beliefs and attitudes for mathematics and mathematics learning amongst students, as perspective goals 'included those aimed at promoting students' ideas and interest in mathematics and learning, such as 'to see that mathematics is fun', and 'to learn to be neat and orderly in their work' (Hollingsworth, Lokan, & McCrae, 2003, p.23).

The broader issue of teacher beliefs and attitudes also has a place in the current discussion, since an individual's emotional rudder (Immordino-Yang & Damasio, 2007) plays a crucial role in guiding his/her decisions and actions. The reporting of more positive teacher affective states arising from participation in sustained professional development programmes in projects such as the ENRP is an indication that teacher affect may well contribute partly to the success of these various projects.

## 2.10 Out-of-school support

There are two key elements in the out-of-school factors that contribute to students' numeracy learning: parents, and private tuition.

### Involving parents in supporting numeracy learning

Many schools in Australia take specific action to involve their parental community in their children's numeracy learning. Goos (2004), for example, noted over 600 specific projects directed at both educating parents, and informing them of the school based approaches.

There is evidence that parental involvement or support is helpful. Cai (2003), for example, investigated the role of parents in the US and in China. He surveyed parents of over 200 US and over 300 Chinese students, and assessed both routine and non-routine problem solving. He investigated five different roles for parents: fostering motivation, providing resources, monitoring progress, advising on content and counselling about learning. Cai concluded that 'parental involvement is a statistically significant predictor of their children's mathematical achievement' (p.87).

Adopting a different perspective, Alexander, Entwistle and Olson (2007), also in the US, compared the achievement of groups of students, focusing on factors that have been identified as contributing to disadvantage, such as SES, and concluded that differences associated with socio-economic factors are attributable to out-of-school learning opportunities, such as participating in extra curricula mathematics classes.

Goos (2004), in Australia, surveyed education providers, professional associations, and parent and community groups, with a particular focus on primary schools, interviewed key decision makers, and conducted case studies. Her framework sought evidence of success, different types of partnerships, perspectives of the respective stakeholders, attention to educational disadvantage, location and lever of schooling. Goos identified key issues in involving parents as recognising different needs and roles, involving parents and community in developments, recognising differences between groups, especially cultural differences, supporting administrators, and finding ways to connect families to schools and informing them of ways they can support numeracy learning. Morony (2004) described one such project, the *Numeracy Research and Development Initiative*, that developed a range of resources to support schools who initiate structured programmes to enlist the support of parents in the numeracy education of their children.

Warren and Young (2002) identified a need for teachers and parents to adopt mutually supportive roles. They noted that parents report their roles as supporting and nurturing their children whereas teachers saw the role of parents as supporting the school programme. The evidence suggests that there is a need to broaden teachers' understanding of the potential benefits of involving parents in numeracy learning generally.

## Out-of-school tutoring

The widespread advertising by out-of-school tutoring companies, including some franchises, suggest that there is demand among parents for additional assistance for their children.

The evidence for the effectiveness of such coaching in improved performance on competitive assessments is limited. Powers and Rock (1999) compared the performance of coached with non-coached students on the high stakes Scholastic Aptitude Testing (SAT) assessments for university entrance in the US. They used careful stratified samples. They reported a small positive effect for coached students on SAT assessment, with more benefit on the mathematics than the verbal components, although they noted that the effect was considerably less than claimed by the coaching companies.

In Australia, Kenny and Faunce (2004), using a range of measures on a large sample of primary and secondary school students, concluded that:

Coached and uncoached students performed equally well in most subjects across most of the academic school years from Year 7 to Year 12...IQ was the best predictor of outcome for all aptitude tests. However coaching had a significant effect on success on (Gifted and Talented) entrance examinations, a lesser impact on entrance to selective high schools, and no impact on scholarship examinations (p.115).

Mak and Mak (2002), noting the disadvantage on comparative assessments experienced by students who did not speak English at home, took a different perspective. They argued that coaching offers parents an option for assisting their children to overcome aspects of their disadvantage.

It appears that the positive effects of coaching might be limited. It is noted, as suggested by Kenny and Faunce (2004) that coaching for competitive assessment might compromise the integrity of the tests, in that they may teach test-taking rather than mathematics, for example. There is anecdotal evidence that coaching for purposes other than for competitive assessments and overcoming some structural disadvantage might have negative effects of receptiveness to schooling instruction.

In summary, it is clear that parents can directly support their children's mathematics learning, and that they are a substantial and perhaps underutilised resource. Out of school tutoring may not be the most effective support to provide for students. A helpful emphasis is likely to be on parents involving students in everyday activities in which mathematics is enhanced. These could include assisting with shopping (e.g. determining the best value of competing items or selecting items of particular sizes – 'please get me the 200g pack'), interpreting maps during family travel, weighing and measuring ingredients during cooking, and participating in board and other games which involve mathematical concepts. It would be useful to find ways to optimise the involvement of parents in the education of their children and to identify the necessary resources for facilitating this.

# Chapter 3: Addressing the numeracy needs of particular groups of students

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## 3.1 Introduction

In Chapter 1, we pointed out that although overall levels of numeracy amongst Australian students is quite sound by international standards, there are large groups of students who do not achieve well. The achievement of a number of countries that do even better than Australia in national assessments such as Finland and Canada, would indicate that the lower achieving students' results in Australia can be improved and that, in human capital terms alone, it is the sensible ambition for a nation with the advantages Australia has.

## 3.2 Indigenous and cultural minority students

The mathematics taught in Australian schools we might call 'Western-techno mathematics'; the 'techno' being short for 'technological' indicating that it has its roots in Western technological society. Steen (1988) describes mathematics as the science of patterns; a way of organising and classifying. Christie (1996) states that 'Mathematics is not a language, nor is it an object. It is a practice: the unseen work done by individuals and groups making sense of their lives, their territories, their histories, and economies through particular discourses which involve naming, ordering, recursion and valuing'. These descriptions remind us that *all* cultural groups seek to understand and make sense of their environment and their practices through identifying patterns that assist in organisation (Perso, 2003).

Western mathematics derives from a western 'world view' which is largely about economics. Quantification dominates, using units and numbers attached for comparison which are powerful elements of trade and negotiation. Measurement enables every square centimetre of the planet to be managed and controlled. This world view pervades the mathematics taught in schools (Perso, 2003).

The mathematics of the Western technological world is filtered by the people belonging to this cultural group. Jones et al. (1995) argue that 'presenting this compartmentalised decontextualised body of Western knowledge to learners with a different world view scheme invites failure for both the learner and the teacher' (p.2). Harris (1989) describes this as the 'wide difference between teachers and pupils in their understanding of the nature of reality and the way they organise the world to find meaning in it' (p.91).

Teachers need to support children from other cultural groups to 'bridge' the cultural divide, not only in the learning of the mathematics taught in schools and numeracy acquisition, but in scaffolding students' home language to Standard Australian English and scaffolding cultural norms, expectations and behaviours in order that students feel included and accepted. At the same time, it is crucial that the rich cultural traditions of the diverse range of students in our classrooms are valued.

### Indigenous education

A deficit view of Indigenous students pervades many school and classrooms, as it does general community commentary in Australia. Often this reflects ignorance of Indigenous cultures and how they manifest themselves. Indigenous students are blamed for their absenteeism, disadvantaged social background and culture. This can result in teachers having low expectations of their Indigenous students' learning

capacities with these expectations producing the very effect they predict. Improving the state of mathematics learning amongst Indigenous students presents perhaps the major equity challenge facing numeracy policy in Australia. It will, however, take a paradigm shift to alter the 'culture of blame', and hence to create a positive framework in which to address Indigenous student achievement in general and in numeracy and literacy in particular [deVries & Warren, 2007 in Independent Schools Queensland, 45]; Perso, 2003).

In response to a national report in 1999 asserting that 'little progress overall has been made in improving the numeracy outcomes of Indigenous students and, in many cases, outcomes for 1999 were below those of previous years' (MCEETYA, 1999, see also Rothman, 2002), initiatives have been implemented within Australian states and territories in an attempt to address this issue for Indigenous students. The pattern of Indigenous achievement continues to mirror that of the rest of the students in Australia in the respective grade levels; the achievement gap between Indigenous and non-Indigenous students does not appear to be narrowing despite the plethora of initiatives targeting Indigenous students' mathematics learning.

The relatively low improvement in Indigenous achievement is reflected in student data across the Tasman. The Numeracy Development Project (NDP) in New Zealand uniquely interpreted student achievement according to socio-cultural factors on top of whole-group comparisons across different years. Through this approach, the overall improvement in student achievements in mathematics as a result of being part of the project (i.e., the NDP) is contextualised within the parallel finding that students of Maori and Pacific Islands descent benefited less than their ethnically Asian and European students through such participation.

Several studies have been carried out to attempt to explain why policies and initiatives aimed at improving Aboriginal students' mathematics achievement often fail. Dawe and Mulligan (1997) highlighted their concern that a high percentage of Indigenous students did not respond to survey items which asked for written answers (compared to those where students needed only to select the right answers). Howard (1997) argued that the imposition of a 'Western' (mathematics) curriculum (see also Cooper, Baturo, & Warren, 2005) to these students has meant that 'for many Aboriginal children ... the mathematics classroom becomes an alien place characterised by tensions and conflicts about relationships and the value of what they are being taught' (p.17). This view is similar to what Aikenhead (2001) described as representative of students' cultural border crossing from their respective cultures and subcultures into that of Western mathematics, a journey that can be difficult to negotiate. Howard and Perry's (2005) work in a remote rural community found that the teaching of mathematics to Indigenous students can sometimes fail to be inclusive and reciprocal. It is thus reasonable to try and understand how Indigenous students continue to have a sense of being outsiders (Tobias, 1988) in mathematics lessons.

Indigenous students' mathematics learning may be enhanced by accounting more for the unique learning styles of such students in their sociocultural context. The way in which Yirrkala Community School in the Northern Territory successfully brings together 'Western' and Yolgnu mathematics in its Gama Living Maths programme exemplifies the value of making mathematics more accessible, connected and meaningful to all students. It could be noted that there is a danger of teachers failing to recognise differences in learning styles, and instead adopting a deficit approach to teaching Indigenous students (Perso, 2003), in so doing perpetuating inequity in the teaching of these students.

More recent projects have benefited from lessons learnt and subsequently reported positive and promising outcomes. The 2006 Mathematics in Indigenous Contexts (K-2) project in New South Wales, for example, was a pilot study in which primary schools and their immediate communities were supported

in developing mathematics learning activities which allowed Aboriginal students to demonstrate their numeracy understanding. Generally, participating students in each of the project schools improved on their pre-test scores on the New South Wales Schedule for Early Number Assessment, and significantly, made greater leaps in this test over the same period when compared to non-Indigenous students (Erebus International, 2007).

This New South Wales project echoed several other projects where the Indigenous community is actively involved in the development of the school mathematics curriculum. Meaney's (2001) work with a Maori school community in New Zealand is one such example, which was driven by a concern for a school mathematics curriculum which is meaningful and relevant to the local Maori community, thus helping to preserve the students' unique cultural identity.

Thus there have been significant improvements at the local level where initiatives target particular groups of students, particular Indigenous groups and their communities. This is appropriate considering the broad range of Indigenous cultural and language groups across Australia. Specifically, programmes implemented indicate improvements in the outcomes of Indigenous students as a result of:

- teachers and school communities valuing the culture, language and the richness of what Indigenous students bring with them to the classroom
- having high expectations of students and their learning (valuing different approaches to learning and recognising and valuing different learning pathways)
- stability of staffing (critical when considering the importance of Indigenous students' relationships with their teacher)
- the importance of teachers building strong relationships with their students
- the use of the community and Indigenous partnerships to create a culturally and contextually aligned learning programme
- the importance of Indigenous educators to bridge the school/community divide
- the critical importance of having first language speakers in the classroom to assist learners to elaborate and scaffold their mathematical thinking
- the use of relevant and meaningful contexts to 'situate' the learning in their lives
- adopting strategies to deal with hearing loss, homework incompleteness, and absenteeism
- teachers paying particular attention to socio-cultural differences in learning styles in the delivery of the mathematics curriculum, for example
- teachers valuing different pathways to learning, (for example use of subitising as a building block for quantifying)
- teachers recognising and paying attention to different teaching and learning styles attributed to cultural differences, for example
  - specifically teaching students to talk about their mathematical thinking, in response to a recognition that many Indigenous cultures use talk primarily for social purposes rather than for teaching and learning
  - supporting Indigenous students to take risks with their mathematics learning and to learn through incremental behaviours, in recognition that many Indigenous students learn, at home, by watching a whole task and then tackling something new when they feel confident of succeeding - thus avoiding feelings of 'shame'



- not directly aiming questions that probe incorrect thinking or publicly drawing attention to errors in recognition that there is potential for Indigenous students to be 'shamed' by peers, thus reducing their disposition to engage in tasks, and that in some Indigenous cultures it is offensive for adults who are not family, to point out errors made by children
- teachers giving instructions and communicating with Aboriginal children using a quiet/gentle approach in recognition that in some Indigenous cultures it is considered offensive to speak forthrightly and strongly.

It should be noted that the above success enablers are equally applicable to teaching and learning programmes and pedagogical strategies for all minority groups since equity is about teacher/student relationships and access to learning through pedagogy.

There has also been evidence that supporting teachers in their professional practice with Indigenous students can yield results. An example in point is the Improving Numeracy for Indigenous Students in Secondary Schools project in Tasmania (Callingham, 1999).

An evaluation study of the Mathematics in Indigenous Contexts (K-2) project arrived at the finding that the most worthwhile support and intervention takes place in the early years of a student's education experience (Erebus International, 2007). Thus, it appears that investment in early childhood and primary school institutions of preventive and interventional educational programmes could pave the way towards motivated and achieving Indigenous students in primary, secondary and tertiary education contexts. This is in addition to known enablers of change in learning organisations, such as top leadership support, an understanding by staff members of the project purposes and expectations, the development and maintenance of a productive working relationship between project managers and participating school staff and community members, and the need for teachers to perceive the initiatives as building on to their current practice rather than additional new work (Erebus International, 2007).

In relation to leadership support, the Mathematics in Indigenous Context (K-2) project in New South Wales has benefited from the professional support given by one member of the New South Wales Board of Studies, as well as from the engagement of Aboriginal education assistants. Real, sustained support from school principals would also increase the long-term impact of any gain – and consolidate change in professional practice – from participating in the initiatives or projects.

Watson, De Geest, and Prestage (2003) found that when student proficiencies, rather than deficiencies, dominated their way of thinking about students, teachers were able to create effective learning communities. For example, in an Australian context, teachers who recognise and value the superior capacities of their Indigenous students in being able to visualise three dimensional spatial relationships and subitising quantities at very early ages in comparison with their non-Indigenous classmates, will likely hold these students in high esteem, value them publicly and create a positive learning environment through the learning expectations made possible through this recognition. They will also recognise and value variable pathways to learning that do not require assimilation of mainstream beliefs and knowledge that may threaten to compromise the cultural identities of their students.

## Socio-cultural factors

Research has often indicated the correlation of SES and (mathematics / numeracy) achievement. For example, in New Zealand, the NDP's interpretation of student achievement data according to socio-cultural factors has revealed that all students benefited from participating in the project, regardless of

ethnicity, gender, and SES. However, while the Asian and European / Pakeha students – as well as those whose schools were located in high socio-economic areas – started off at higher framework stages, what deserves greater attention is that these groups of students achieved greater gains than their peers. That is, ‘the project did not narrow the ‘achievement gap’ as hoped, but instead widened the gap slightly’. Similarly, an analysis of the TIMSS 1994 data failed to contradict the assertion that given any curriculum, students from high SES background performed better than their peers (Hook, Bishop, & Hook, 2007). Rothman’s (2002) analysis of mathematics scores of Australian 14-year-old students between the period 1975 and 1998 supported this trend, with her adding that the differences between attainment measures were statistically significant. Interestingly, over this period, when the mathematics scores of students from professional/managerial families became wider in distribution, those of students from labourers and related workers families actually registered a narrower range.

Hook, Bishop and Hook’s (2007) five-year study involving two cohorts of students (totalling more than 13,000) revealed that, given the introduction of a ‘quality curriculum’, economically disadvantaged and immigrant students can show substantial improvements in their mathematics learning. On the other hand, analysis of the PISA 2003 achievement data for Australia showed that there was no significant difference in mathematical literacy regardless of the immigrant status of the students, that is, native (Australian-born), first-generation, or non-native (foreign-born) (Thomson, Cresswell & de Bortoli, 2004). The distribution of proficiency levels attained by these groups of students was also similar.

Indeed, the variability of student characteristics within immigrant student populations would restrict the meaningfulness of research studies into these students as a group. To this end, the immigrant status of individual students might be differentiated according to the main language spoken at home. With this perspective, a differentiation amongst immigrant students’ performance can be made. For example, the Australian students whose language background was English ‘performed at about one-quarter of a standard deviation above the OECD average, while those with a language background other than English performed at around the OECD average’ (Thomson, Cresswell & de Bortoli, 2004, p.88).

## **Numeracy, language and context**

As was indicated in Chapter 2, language has a central role to play in the development of numeracy. The decade or so before the turn of this millennium saw substantial research into how students’ language abilities might impact on mathematics learning (see, e.g., Ellerton, Clements, and Clarkson, 2000). Rothman (2002) examined the achievement data of 14-year-old Australian students across the years 1975 to 1998, concluding that while mean scores for mathematics of students from homes where the main spoken language was English remained higher than students who spoke other languages at home, the gap in these mathematics mean scores was narrowing. However, over the last few years, research into language factors in mathematics education has been rather sparse. In fact, Zevenbergen (2000) concluded that there had been little knowledge in any systematic way of the impact of language on the numeracy growth of primary school students. Similarly, there is limited research being conducted to assess or evaluate related programmes in the classroom.

The number of immigrant teachers of mathematics is a small but increasing proportion of the Australian teacher workforce. Seah’s (2004) doctoral thesis revealed that many immigrant teachers found that contrary to their initial beliefs that mathematics is the same in different countries, the way in which it is taught can and does differ from country to country. As a result, immigrant teachers of mathematics experience cultural value difference and dissonance. A variety of negotiation approaches were adopted



by the teacher participants in the study. In this light, given the absence of any orientation programme for immigrant teachers in Victoria, Seah and Bishop (2006) argued that there are implications for the ways in which mathematics is taught in the Australian classroom.

In Chapter 2, the role of and understanding of context on numerate behaviour was described. Clearly, what is familiar and unfamiliar can have a significant impact both on a student's capacity to learn mathematics from a particular context and on his or her capacity to demonstrate learning. The context in which a mathematical problem is set has the potential to disadvantage those who are unfamiliar with it. Zevenbergen (2001) contended that children's familiarity with aspects of language are related to their socio-economic backgrounds and this could also affect mathematical performance. Developing the capacity to become and be numerate in a range of contexts requires not only that students are able to link mathematics to their own experience of life but also that they know how to do so and how not to do so (Willis 1998). But this is no easy or obvious matter, certainly it demands more than the so-called 'story problems' or 'applications' of mathematics as this wonderful and now well-known example from the US shows.

A Ute student was asked to determine how much his brother would have to spend on gasoline if he wanted to drive his truck from the reservation to Salt Lake City. Instead of estimating (or generalising) a response, or attempting to calculate an answer based on the information presented in the request, the student responded quite simply:

'My brother does not have a pickup' (Leap 1988, p.176).

In keeping with his cultural tradition, this student balked at a discussion not grounded in truth.

Ute philosophy takes precedence over non-Indian perspectives in other areas of daily life, so why should Ute philosophy not take precedence over the content of classroom instruction (p.177).

It is a characteristic feature of mathematics that we idealise reality and focus only on certain features of real situations. It is the basis of mathematical modelling that we do so. Indeed, an important part of learning mathematics is to learn to play the 'lets pretend' game. However, for many students this may not be an obvious or an easy matter.

In teaching mathematics to Indigenous students who have English as their second language (ESL) or who speak Aboriginal English as a dialect of spoken English (ESD), there is a critical need to recognise the importance of spoken language as the foundation of all learning. If the discourses of the home do not match those of the school then this is known to disadvantage Indigenous students' achievements in literacy and numeracy in the long term (Dickinson, McCabe & Essex, 2006; DEST, 2006a).

The home language of Indigenous students must connect with the underlying meaning of mathematical concepts. Teachers should not assume that Indigenous students will share their understanding of the English words and concepts that they use. In some traditional/non-urban or rural Indigenous contexts for example, numbers may be familiar to students in a *nominal* sense through everyday contexts such as numbers on vehicle number plates or football jumpers, but not in a cardinal sense, such as for comparing quantities (e.g. I have 20 pens and he has 25). Students may need to be explicitly taught the use of numbers for comparison to provide a context for learning how to count to determine 'how many'.

The language of *position* (words such as *under, behind, on, near*) is critical in early mathematics lessons about Space. These words are familiar to most non-Indigenous students who in general are immersed in this language in the home well before commencing school. In many Indigenous households these words

are not used since gestural language is used. Both approaches are appropriate for the children in their homes and communities. However, the spatial terminology used in non-Indigenous homes advantages students in accessing the mathematics curriculum in Australian schools. Teachers need to bridge this cultural divide through explicit teaching of the words, not assuming they are known and judging students for not knowing them and treating them as deficient. Since this language is particular to the school context for some students, students will need to use this language (at least initially) as 'school talk', being given permission to 'code switch' between school and home as appropriate. This approach helps to support the students' unique cultural identity.

### 3.3 Interventions to support underperforming students

Various programmes – including national benchmark tests – identify students who are underperforming. While recognising that there is some contention about it, the term 'at risk', is used here to refer to such students.

It seems clear that there are students who experience difficulty in learning, and without specific teacher interventions are at risk of longer term underperformance. Gervasoni (2004), for example, found that by the end of the first year at school, some 40% of students are at risk in at least one aspect of number learning. The number and combinations of domains for which children are at risk is diverse, highlighting the complexity involved in assisting them.

Gervasoni (2004) argued that at risk students can lose confidence in their ability, and develop poor attitudes to learning and to school. One outcome is that the gap grows between the knowledge of these children and of other children, and the 'typical' learning experiences provided by the classroom teacher for the class do not enable each child to fully participate and benefit. Ginsburg (1997) concluded that 'as mathematics becomes more complex, children with mathematics learning difficulties experience increasing amounts of failure, become increasingly confused, and lose whatever interest and motivation they started out with' (p.26).

This issue is of concern internationally. The *Guardian* of May 15, 2007, reports an initiative from the incoming UK Prime Minister to 'find funds to ensure that by 2010 more than 300,000 at-risk pupils a year benefit from one-to-one tuition in maths, with 30 to 40 hours a year for those with greatest need' (Wintour and Meikle, 2007).

There are two strands identified in the literature for addressing the needs of at risk students: one is based on structured withdrawal programmes; and the other involves addressing students' needs in mainstream classrooms.

#### Numeracy learning withdrawal programmes

There are several programmes that address the challenge of 'at risk' students through withdrawal programmes. The *Mathematics Recovery Programme* (Wright, Martland, & Stafford, 2000), for example, engages children in the second year of schooling who have been identified as at risk, in long term individualised teaching with the aim of advancing the students' arithmetical learning to return to classroom. It is a one-on-one withdrawal programme that involves identification, after one year of school, of low achieving students apparently unable to benefit from classroom mathematics teaching. These low attaining students take part in an intensive, individualised teaching programme aimed to advance them to an average level. This programme has been adapted and is being used in the US. Cobb (2005), for

example, listed the aims of the US programme as providing 'a robust intervention framework for teachers working with elementary students to help in the construction of numeracy skills, through assessment which incorporates a strong analysis component and individualized teaching'. A 'structured and objective assessment system that allows educators to know exactly where students are in their mathematical development and apply early, short term intervention' (p.3) is used. Cobb reported that approximately 75% of *Math Recovery* students reach the average level of performance in 10 to 15 weeks.

Similarly, *Mathematics Intervention* (Pearn & Merrifield, 1998) is a withdrawal programme for small groups of students in their second year of school, offering children of similar ability a chance to participate before they experience long term failure. Having identified children as being at risk, trained teachers emphasise verbal interactions between teacher and children and between children, with the goal of building student understanding.

Another example in Australia, the *Extending Mathematical Understanding Programme* (EMU, Gervasoni, 2004), is an intervention programme designed for 6- and 7-year-old children who are at risk in aspects of number learning. It aims to enhance and accelerate children's number learning, and prepare children to benefit more fully from the regular classroom mathematics programme. The EMU Programme comprises daily 30-minute sessions for between 10 and 20 weeks. Specialist teachers work with groups of three students. This process has now been extended for use across the primary school (K to 6).

Gervasoni (2004) devised a process for identifying children at risk and for prioritising children for participation in the intervention programme. Also identified by Gervasoni were common errors and difficulties in number learning that are useful for teachers to know about, and that can be used as a focus for planning instruction, and for teacher professional development. In 2000, the effectiveness of both small group and individual programme structures were trialled, with small groups found to be most effective. Gervasoni argued that the EMU Intervention Programme provided children with a different level of interaction with the teacher than is possible within the classroom setting during mathematics lessons. Observations of more than 30 EMU sessions in 2000 showed that, within each 30-minute session, children and teachers engaged in more than 100 interactions focused on the mathematical ideas investigated during a session.

Connected to all of these withdrawal programmes is an approach to instruction for at risk students. Ellis (2005), for example, in a review of the psychological literature on teaching students with learning difficulties, argued that students experiencing difficulty should be given direct instruction. Ellis emphasised explanations, including scripted presentations, teaching essentials, and small group instruction, and she recommended rapid pacing and drill. She argued that direct instruction is significantly more effective for mathematics teaching than what she termed constructivist instruction.

Similarly, Pegg, Graham and Bellert (2005) described a study with low achieving 11 to 13 year olds that included explicit teaching of number facts in regular and extended sessions over 25 weeks, focusing on quick response ('quick') and appropriate strategies ('smart'), as part of the *Quicksmart* programme. They reported that students improved in their speed of recall of number facts, and improved on state wide assessments, even after a 12-month delay. They also reported sustainable gains in students' achievement up to two years.

While these approaches are resource intensive, there is documented evidence of their success.

## In classroom support

There are also programmes that provide support for at risk students within mainstream classrooms. A programme in the US, *Mathwings* (Madden, Slavin, & Simons, 1999), places emphasis on prevention and on the importance of the regular classroom programme, with the aim of improving the teaching of all teachers so that children experience success in the mainstream.

Similarly, *Getting it Right* is a West Australian literacy and numeracy initiative (Department of Education and Training Western Australia, 2007) providing for the training and placement of specialist teachers to assist in diagnosing the needs of students who are at risk and provide programmes that meet their needs. In nearly 250 schools in 2004, certain groups whose levels of literacy and numeracy lagged behind those of the general population were targeted in particular: Aboriginal students; boys; students with a language background other than English, and students in rural and remote locations.

The website explains:

Specialist Teachers work shoulder-to-shoulder with classroom colleagues, collaborating with them in the classroom, modelling integrated teaching strategies in their area of specialisation and supporting the planning and implementation of effective teaching and learning programmes. They do not routinely withdraw groups of students from a class, and the classroom teacher maintains responsibility for the progress of all students in the class. Specialist Teachers share their expertise with colleagues and gradually build the capacity of the whole school to improve literacy and numeracy. They support the collection, analysis and use of information about literacy and numeracy progress of individual students, groups and the whole school so planning decisions can be informed by quality evidence of learning and ongoing needs.

Factors attributable to the success of the *Getting it Right* strategy include:

- the use of student data by classroom teachers to set targets for improvement and monitor achievement
- a whole school approach
- the development of teacher-pedagogic content knowledge through on-going professional development
- the focus on pertinent key understandings of students
- high teacher expectations of all students
- the de-privatisation of teacher practice
- the use of specialist teachers in the school community.

The Archdiocese of Canberra and Goulburn Catholic Education Office has identified the following as success factors in raising the numeracy outcomes of 'at risk' students:

- early identification of students experiencing difficulties
- intensive monitoring and programming by Learning support teachers in conjunction with all stakeholders
- high quality professional development for all staff
- targeted appropriate funding for 'at risk' students [Archdiocese of Canberra and Goulburn Catholic Education Office, 44].

Similarly, Catholic Education South Australia identifies the use of written effective feedback which supports student learning by:

- affirming what the student knows and has done that is useful for their learning
- providing opportunities to reflect on understandings and seek clarification of possible misunderstandings
- setting possible effective directions for subsequent learning (William, 2005).

They highlight the effectiveness of this approach in improving the enthusiasm and attitude towards mathematics of at risk students – a factor they saw as essential for creating a belief by the students of their ability to learn mathematics.

### **Recommendation 6:**

To raise the overall level of achievement, increased resources (including specialist teachers working 'shoulder to shoulder' with teachers) should be directed to support teachers in regular classrooms to provide intervention for a higher proportion of students during all the compulsory years of schooling, and that:

- the focus of intervention for students at risk be on enabling every student to develop the in-depth conceptual knowledge needed to become a proficient and sustained learner and user of mathematics; and
- these resources be particularly focused on the early years of schooling.

Adopting a similar approach, Sullivan, Zevenbergen and Mousley (2005) conducted detailed research in classrooms to examine whether, given appropriate resources, teachers could support the learning of at risk students. They found that it was possible to create sets of experiences that had the effect of including all students in productive learning, including students experiencing difficulties in learning. Their model of planning and teaching included attention to the choice of tasks and their sequence, specific prepared prompts for at risk students, and the building of an inclusive classroom community. Sullivan et al. (2005) argued that if classroom programmes assume that mainstream students are learning from constructive activity, it does not seem logical that students experiencing difficulty would learn better by listening to explanations. At least in part, this is because different treatment can result in different expectations. Brophy (1991), for example, described the negative effects of self-fulfilling prophecies. Brophy argued that rather than grouping students by their achievement levels, teachers could: concentrate on teaching the content to whole class groups; keep expectations for individuals current by monitoring progress carefully; let progress rates rather than limits adopted in advance determine how far the class can go; prepare to give additional assistance when it is necessary; and challenge and stimulate students rather than protecting them from failure or embarrassment.

Dweck (2000) argued that finding ways to support students at risk is as much connected to their orientation to learning and cognitive approaches. Dweck categorized students' orientation to learning in terms of whether they hold either mastery goals or performance goals. Students with *mastery* goals seek to understand the content, and evaluate their success by whether they feel they can use and transfer their knowledge. They tend to have a resilient response to failure, they remain focused on mastering skills and knowledge even when challenged, they do not see failure as an indictment on themselves, and they believe that effort leads to success. Students with *performance* goals are interested predominantly in

whether they can perform assigned tasks correctly, as defined by the endorsement of the teacher. Such students seek success but mainly on tasks with which they are familiar, they avoid or give up quickly on challenging tasks, they derive their perception of ability from their capacity to attract recognition, and they feel threats to self worth when effort does not lead to recognition.

Similarly, Watt (2004) argued that course choices and achievement are related to students' self-perceptions, including their rating of their ability, and their expectations of success, the value they attribute to the particular content, such as its intrinsic value and its usefulness, and their evaluations of a particular task, such as its difficulty and the amount of effort required to complete it. Similarly, Martin and Marsh (2006) described adaptive or helpful characteristics of students' orientation to learning as the extent to which they feel they can succeed at a task, their valuing of school, mastery orientation, persistence, planning and self management. Connected to this is the extent to which students' connect current schooling with future opportunities or their possible selves, which is 'the future-oriented component of self-concept' (Oyserman, Terry, & Bybee, 2002, p.313).

In summary, there is evidence of successful approaches to supporting at risk students through withdrawal programmes, both individually and in groups and involving direct teaching. There is also evidence of successful approaches that support at risk students within classrooms. Classroom approaches seem to have the added benefits of supporting teacher professional development and therefore building school capacity. The classroom based approach increases teachers' understandings of student learning and of individual student needs. The classroom based approach promotes differentiated pedagogies, thus increasing the equity of learning.

In particular, programmes which are successful with students at risk and those from varying cultural and language background appear to share certain features:

- teachers and school communities valuing the richness of culture and language that students bring with them to the classroom
- high expectations of students and their learning
- stability of staffing which is particularly critical when considering the importance of students' relationships with their teacher
- the use of the community partnerships to create a culturally and contextually aligned learning programme with Indigenous and other minority culture educators to bridge the school/community divide
- the presence of first language speakers in the classroom to assist learners to elaborate and scaffold their mathematical thinking
- the use of relevant and meaningful contexts to 'situate' the learning in students' lives
- strategies to deal with particular issues such as hearing loss, homework incompleteness, and absenteeism
- valuing different pathways to learning including by (a) specifically teaching students to talk about their mathematical thinking (e.g. many Indigenous cultures use talk primarily for social purposes rather than for teaching and learning) and (b) supporting students to take risks with their mathematics learning.

These *success enablers* are equally applicable to teaching and learning programmes and pedagogical strategies for *all* groups – equity being about teacher/student relationships and access to learning.

**Recommendation 13:**

That all teachers of mathematics and numeracy be equipped to identify and understand how personal circumstances, cultural practices and the particular mathematical needs of individual students may impact upon their learning of mathematics, and to intervene as necessary, drawing on a repertoire of effective pedagogies to ensure that these learning needs are met.



# Chapter 4: Teacher education and professional development

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## 4.1 Introduction

This chapter discusses research and submissions on mathematics pedagogical content knowledge of prospective and in-service teachers. It explores the relationship between teachers' knowledge of, and confidence with, mathematics, their mathematics content knowledge, their practice, and the impact of these on student numeracy outcomes.

In Chapter One there is considerable discussion around an apparent lack of suitably qualified teachers, particularly in mathematics. It is suggested that there is a significant number of teachers who are teaching mathematics in the secondary school without undergraduate training in mathematics. Documents such as the *Australian Council of Deans of Science Report* (ACDS, 2006) and the *National Strategic Review of Mathematical Sciences Research in Australia* (Australian Academy of Science, 2006) present a well argued case that the future availability of well qualified mathematicians is under threat unless greater emphasis is placed on mathematics at all levels including school [AMSI, 49].

It is noted that there is a complex relationship between the level of mathematical studies and the capacity to teach mathematics well. For example, while perhaps it is not essential that prospective primary teachers study calculus as part of their teacher preparation, it is clear that there is connection between the relevant mathematics they do know and their capacity to teach. One of the challenges is to identify the knowledge required for teaching, how this is best developed, and the interplay between knowledge, beliefs and practice. Teachers need robust content knowledge to enable them to support, direct and guide their students.

In this section the focus is specifically on teachers of mathematics – those with specific mathematics curriculum responsibility – at either primary or secondary level. However, most of the literature that has been identified to date involves research with primary teachers.

## 4.2 Mathematical knowledge for teaching

In this section the types of knowledge that are required for the teaching of mathematics and research on the impact of teachers' mathematics knowledge on student achievement are discussed. Some examples are provided as illustrations. The link between beliefs and attitudes in the teaching of mathematics and the place of teachers' confidence and strategies for developing this are also discussed.

There is clear evidence on the relationship between knowledge and teaching. Darling-Hammond (1997) summarised research on data from 900 school districts in Texas that found that 40 per cent of the measured variance in student achievement across Grades 1 to 11 was due to teacher expertise. Her main measures include students' scores on mathematics and literacy assessments. She argued that, even after controlling for socio-economic status, the large differences in achievement between 'black and white [students] were almost entirely accounted for by differences in the qualifications of their teachers' (p.8). Kilpatrick, Swafford and Findell (2001) reported that 'there seems to be no association, however, between how many advanced mathematics courses a teacher takes and how well that teachers' students achieve overall in mathematics' (p.324). They go on to suggest:

That crude measures of teacher knowledge, such as the number of mathematics courses taken, do not correlate positively with student performance data, supports the need to study more closely the nature of the mathematical knowledge needed to teach and to measure it more sensitively (p.375).

It is important to focus on forms of knowledge closely related to teaching, in particular *pedagogical content knowledge* (Shulman, 1987). There is increasing agreement that the mathematical content knowledge required for teaching is connected to the teaching of particular content, for example, fractions, and that *how* teachers hold knowledge may matter more than *how much* knowledge they hold (Hill & Ball, 2004). It has been argued that teachers need to be able to deconstruct their own mathematics knowledge into less polished and final forms, where elemental components are accessible and visible. 'Because teachers must be able to work with content for students in its growing, not finished state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to unpack its constituent elements' (Ball & Bass, 2000, p.98).

Ma (1999), in her research study of the differences between the mathematical knowledge for teaching of US and Chinese primary teachers, identified four facets of teachers' mathematical knowledge which are crucial to teachers' 'profound understanding of fundamental mathematics' (p.122). These are knowledge of basic mathematical ideas, connectedness, multiple perspectives or representations, and longitudinal coherence ('fundamental understanding of the whole elementary curriculum'). One of her key findings was the greater ability of Chinese teachers to perform mathematical tasks at the elementary level, while the American teachers who had typically undertaken more tertiary study displayed less subject matter knowledge and pedagogical content knowledge than their Chinese counterparts.

Hill, Rowan and Ball (2005) focused their study on *mathematical knowledge for teaching* and developed an instrument for assessing this specifically. They found a positive correlation between the teachers' mathematical knowledge for teaching and student achievement among the Grade 1 and 3 teachers who were studied (115 schools over a four-year period). One of the difficulties with any research in this area is that teaching is complex, and separating the impact of other variables is challenging. Hill et al. (2005) were able to control for a number of variables and also looked at the number of mathematics related units that teachers had undertaken during their teacher preparation. Their findings suggested that, at the junior primary level, increasing teachers' subject matter or methods course work does not ensure 'a supply of teachers with strong content knowledge for teaching mathematics' (p.393).

The demands on teacher understanding have increased as educational practices change to emphasise the concepts behind content, rather than the earlier focus on procedural knowledge. Brophy (1991) argued in relation to content knowledge that:

where (teachers') knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways and encourage and respond fully to students' comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasize interactive discourse in favour of set work assignments, and in general, portray the subject as a collection of static, factual knowledge (p.352).

Clearly, the way knowledge is organised and accessed as well as the nature of that knowledge is important. It must also be acknowledged that in many countries (including Australia) there has been a shift in focus from a transmission model of teaching to an emphasis on teaching for understanding (Fennema & Romberg, 1999). It is no longer a case of the student 'working out what is in the teacher's head' but, instead, on teaching that aims to understand and build on what the student is thinking.

Moving to a more learner-centred approach places greater demands on teacher knowledge as the lesson can take many possible directions; given the more responsive nature of the teaching process, students' strategies and reasoning could well challenge the teacher's mathematical 'comfort zone.'

There is increasing evidence that the provision of knowledge based on students' thinking (Carpenter & Lehrer, 1999), particularly for teachers in the early years of schooling, is contributing to improved teaching practice and student outcomes. The use of research based frameworks and assessment which inform teaching practice, such as those in the ENRP and CMIT (see Bobis et al., 2005), have contributed importantly to the improvements in student learning. This builds further on the evidence provided by the Cognitively Guided Instruction Project in the US which:

provides strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. These findings, when viewed in conjunction with those of other studies, provide a convincing argument that one major way to improve mathematics instruction and learning is to help teachers understand the mathematical thought processes of their students (Fennema et al., 1996, p.432).

Baturo et al. (2004) found that:

Students' numeracy outcomes were enhanced when teachers' pedagogic knowledge incorporated a theoretical framework that enabled them to plan and implement units that focused on the development of structural knowledge. Such knowledge took into account appropriate sequences, connections, task, talk, and generic strategies, as well as how students comprehend, misconstrue and forget (p.xviii).

To illustrate the nature of the mathematical knowledge that is important for primary teachers, the following is a sample item assessing teachers' appreciation of aspects of fractions:

A teacher has asked the students to determine the larger of two fractions for each of the following pairs:  $\frac{3}{4}$  and  $\frac{3}{6}$ ;  $\frac{1}{2}$  and  $\frac{1}{3}$ ;  $\frac{5}{7}$  and  $\frac{5}{9}$ , and to explain their thinking. It is likely that the teacher chose this set of fraction pairs so that students may understand that (please choose all alternatives that apply, explaining your decisions below):

- a. The smaller the gap between the numerator and denominator, the larger the fraction
- b. Converting to common denominators is essential in order to compare fractions
- c. When two fractions have the same number of parts, you need to compare the size of the parts
- d. To find the larger fraction, you add the numerator and denominator, and the smaller the sum the larger the fraction

An example of the mathematics required for teachers in the middle years is the study of integers and the understanding of different models and representations. Knowing the rules for computing with integers is insufficient for understanding operations with numbers less than zero. As with whole-number operations, teachers and children must learn to think of the variety of citations that can be modelled by operations. For example, using the model of a lift has limitations. It allows the children to think about subtracting a positive integer as 'going down' or about subtraction as the distance between floor (difference) but the representation does not help the student develop a sense of 'taking away' numbers less than zero. Nor could they make sense of certain addition expressions.

In discussing teachers' knowledge, it is important to acknowledge also the link between knowledge and attitudinal views of teachers and their impacts on students. There is considerable evidence (e.g. DEST, 2004, Baturu et al., 2004) that primary school teachers' confidence and competence with mathematics are a cause for concern.

It is important that the mathematical knowledge of primary teachers is valued. A lack of confidence in mathematics can be due to a lack of knowledge but this is accentuated when others including policy makers appear to value, and expect primary teachers to have, competence at high level mathematics. It is important to describe what mathematics effective primary teachers need to know and use in sophisticated ways.

### 4.3 Pre-service teacher education and prerequisite knowledge

There is sufficient concern and interest in pre-service mathematics teacher education that the Australian Council of Deans of Science produced a report in 2006 titled *The Preparation of Mathematics Teachers: Meeting the demand for suitably qualified mathematics teachers in secondary schools*. There is also considerable research being undertaken within Australia by mathematics teacher educators into issues relating specifically to pre-service teachers' mathematical knowledge. (Goos et al., in press)

While there might be some debate about the extent and form of the mathematical knowledge required to teach effectively, there is clearly a considerable body of knowledge that is required by prospective teachers of mathematics.

There is an important distinction between the development of teachers in the contexts of primary and secondary schools. Primary teachers are generalists who teach most if not all the curriculum areas to a specific group of children, while secondary teachers tend to be subject specialists.

The *Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences, 2001) from the US provides recommendations on the mathematics content requirements for teachers according to the levels at which they are teaching. However, the point is clearly made that:

'this is not to say that prospective teachers will be learning the mathematics as if they were nine-year-olds. The understanding required of them includes acquiring a rich network of concepts extending to the content of higher grades; a strong facility in making, following, and assessing mathematical argument; and a wide array of mathematical strategies' (Chapter 3, p.3).

For example, the authors recommend that to be prepared to teach *arithmetic* for understanding, elementary school teachers (Grades K to 5) need to understand:

- 'A large repertoire of interpretations of addition, subtraction, multiplication and division, and of ways they can be applied.
- Place value: how place value permits efficient representation of whole numbers and finite decimals; that the value of each place is ten times larger than the value of the next place to the right; implications for this for ordering numbers, estimation, and approximations; the relative magnitude of numbers.
- Multi-digit calculations, including standard algorithms, 'mental math,' and non-standard methods commonly created by students: the reasoning behind the procedures, how the base-10 structure of numbers is used in these calculations.

- Concepts of integers and rationals: what integers and rationals (represented as fraction and decimals) are; a sense of their relative size; how operations on whole numbers extend to integers and rational numbers; and the behaviour of units under the operations' (CBMS, 2001, Chapter 3, p.18).

Other areas of mathematics discussed are algebra and function, geometry and measurement, and data analysis, statistics and probability. This document also had suggested content for middle school (Years 7 to 8) and high school (Years 9 to 12), but at these levels the relevance to the Australian context is limited, as both the curriculum and the school structure are different.

The recent review of primary numeracy research commissioned by DEST (Groves et al., 2006) identified a number of studies relating to pre-service teachers. Their findings indicated that:

- there is a strong correlation between student teachers' levels of mathematics performance and their levels of self-confidence
- student teachers often hold beliefs about mathematics and learning that constrain their access to rich and powerful ways of learning and teaching
- many students in teacher education programmes believe that calculator use should be avoided in primary mathematics
- student teachers appear to have had little past experience with activities that might promote number sense or reflection on mathematical processes
- many pre-service teachers believe they are insufficiently prepared in terms of mathematics content, pedagogy and pedagogical content knowledge, but believe they are sufficiently prepared in terms of their knowledge of mathematics curriculum (p.203–204).

One of the debates in primary mathematics teacher education is how the mathematics content is best enhanced in pre-service teacher education – through the context of teaching mathematics or as stand alone mathematics content? There is limited evidence to enable strong recommendations, however as previously mentioned, there is some emerging evidence particularly for primary teachers of the value of studying student thinking to develop both mathematical content and pedagogical content knowledge. (Sowder, 2007)

There is also emerging evidence of the value of focused teaching experiences including the careful study of individual lessons, shared reflections and observations. The *Primary Numeracy* report (Groves et al., 2006) identified innovative practices that were found to be effective in mathematics teacher education. These included:

- a school-based programme in which students took responsibility for teaching a small group of children throughout the year, with findings supporting the long standing psycho-dynamic theory that powerful emotional experiences involving practice and reflection are required if significant and effective change is to occur
- the use of an interactive multimedia resource based on a close analysis of one lesson used to support student teachers in their study of teaching, resulting in pre-service teachers demonstrating increased observation skills as well as improved ability to discuss teachers' work in post-practicum discussions.

Much of the recent Australian research focuses on the preparation of primary teachers and involves small scale studies where mathematics teacher educators study the knowledge, practices and beliefs of their own student teachers. There is little comprehensive and extended study within Australia and Goos et al. (in press) have identified the need for larger scale studies, including longitudinal, cross national and policy studies.

A 2001 report commissioned by the US National Research Council Committee on Science and Mathematics Teacher Preparation focused on teacher preparation across the K to 12 grade level range and recommended that programmes have the following features (in this case, science and technology are taken out of the reference):

- be collaborative endeavours developed and conducted by mathematics, education faculty, and practising K to 12 teachers with assistance from members of professional organisations and mathematics-rich businesses and industries
- help prospective teachers to know well, understand deeply, and use effectively and creatively the fundamental content and concepts of the discipline that they will teach
- unify, coordinate, and connect content courses in mathematics with methods courses and field experiences
- teach content through the perspective of methods on inquiry and problem solving
- present content in ways that allow students to appreciate the applications of mathematics
- provide learning experiences in which mathematics is related to and integrated with students' interests, community concerns, and societal issues
- integrate education theory with actual teaching practice, and knowledge from mathematics teaching experience with research on how people learn mathematics
- provide opportunities for prospective teachers to learn about and practice teaching in a variety of school contexts and with diverse groups of children
- encourage reflective inquiry into teaching through individual and collaborative study, discussion, assessment, analysis, [and] classroom-based research and practice
- welcome students into the professional community of educators and promote a professional vision of teaching (cited in Sowder, 2007, p.200).

These clearly present a model of connected and integrated learning and stress the importance of the mathematics content knowledge being connected to pedagogical content knowledge.

While it is important to discuss the quality and expectations of teacher education programmes in mathematics, of considerable concern are the large numbers of secondary teachers who have no direct preparation for teaching mathematics. The Australian Council of Deans of Science report focused on secondary teachers and used questionnaires that were distributed throughout Australia. Responses were received from 2,924 teachers and 612 heads of mathematics (30% overall response rate) showing that one-third of the junior and middle grades mathematics teachers had not studied any mathematics teaching method.

The report also found some disturbing data on the shortage of secondary mathematics teachers.

- Three in four schools reported difficulties recruiting suitably qualified mathematics teachers. Schools received numerous applications for advertised positions but few applicants had the necessary mathematics background to teach mathematics, particularly at senior school level.
- Schools in more remote regions reported the greatest difficulty. Among the large eastern states, recruitment was a particular challenge for Queensland schools.
- The shortage of available mathematics teachers was seen as a relatively recent and growing problem, predicted to worsen as experienced teachers retire in coming years.

An interesting finding was that early career teachers were more likely than their more experienced colleagues to have been employed in a different industry prior to taking a teaching position, suggesting that career-switching is an increasingly common pathway for mathematics teachers. Half the teachers with less than five years of teaching experience had taken such a path. These teachers were more confident that they would continue teaching than were their 'first profession,' early career peers. This may have implications for teacher recruitment and their training and development.

While the pre-service preparation is important, along with stronger teacher preparation, induction that includes mentoring and further professional development can reduce attrition and at the same time strengthen teachers' abilities to be effective (Sowder, 2007).

### **Assessing pre-service teachers' mathematical knowledge**

While there is considerable discussion about the need to assess pre-service teachers' mathematical knowledge, there seems to be little agreement on what to measure and how. Pressing questions – 'such as the balance of knowledge of content and knowledge of pedagogy, the nature of content knowledge useful for teaching, and the 'content' of pedagogical knowledge – have not been answered' (Hill et al., 2007, p.149). While the push for accountability is not likely to reduce, the challenge is to create the best tests possible. Hill et al. in their recent review of research suggest the following:

- measure mathematical knowledge for teaching – valid teacher assessment should not be remote from what teachers do in the classroom
- measure with care – recognising the advantages and disadvantages of different assessment formats
- use multiple approaches – to enable comprehensive appraisals
- meet professional standards of rigor in assessment – including validation of the results in terms of impact on students
- learn from other measurement methods – more cross-over needs to occur between quantitative and qualitative researchers
- attend to issues of equity
- investigate the relationship among mathematical knowledge for teaching, other domains of teaching knowledge, and student learning
- increase professional role and control.

While the researchers' focus and policy framework is based on the US, the suggestions above provide a useful direction for the Australian context.

There are many challenges in the pre-service education of mathematics teachers and many areas where the research knowledge is limited. However, it is clear that prospective teachers need the opportunity to study mathematics for teaching, the opportunity to study students' mathematical thinking and learning in context, constructive connections between the theoretical knowledge and the practical experience and that teacher education programmes need to allocate sufficient numbers of units to ensure that this content is effectively covered.



## 4.4 In-service teacher education

In addition to initial teacher preparation, as with any profession, ongoing professional learning is vital. Knowledgeable teachers are the key and teaching improvement and increased student achievement depend on the ongoing professional development of teachers.

Sowder (2007), in a recent review of research on the mathematical education and development of teachers stressed the need for teachers to have an opportunity for sustained and serious learning of curriculum, students and teaching, suggesting six goals for professional development:

1. Developing a shared vision for mathematics teaching and learning.
2. Developing mathematical content knowledge.
3. Developing an understanding of how students think about and learn mathematics.
4. Developing pedagogical content knowledge.
5. Developing an understanding of the role of equity in school mathematics
6. Developing a sense of self as a teacher of mathematics.

### Professional teaching standards in mathematics

There are some similarities for the above with the work on professional teaching standards funded by the ARC and the AAMT. This work involved groups of teachers from state associations developing standards for highly accomplished teachers of mathematics, with input from a range of stakeholders. They identified three domains in describing accomplished teachers of mathematics, representing goals for teacher development and learning (AAMT, 2006; <http://www.aamt.net.au/standards>) as follows:

#### Professional Knowledge

Excellent teachers of mathematics possess a strong knowledge base in all aspects of their professional work including their decision making, planning, and interactions. This includes knowledge of students, how mathematics is learned, what affects students' opportunities to learn mathematics and how the learning of mathematics can be enhanced. It also includes sound knowledge, training, and appreciation of mathematics appropriate to the grade level and/or mathematics subjects they teach.

#### Professional Attributes

Excellent teachers of mathematics are committed and enthusiastic professionals who continue to extend their knowledge of both mathematics and student learning. They work creatively and constructively within a range of 'communities' inside and beyond the school and set high, achievable goals for themselves and their students. These teachers exhibit personal approaches characterised by caring and respect for others.

#### Professional practice

Excellent teachers of mathematics are purposeful in making a positive difference to the learning outcomes, both cognitive and affective, of the students they teach. They are sensitive and responsive to all aspects of the context in which they teach. This is reflected in the learning environments they establish, the lessons they plan, their uses of technologies and other resources, their teaching practices, and the ways in which they assess and report on student learning.

In addition to the use of the teaching standards in a regulatory way for evaluating and credentialing, there is increasing evidence of the value of these for professional development purposes including assisting individuals and groups of teachers to identify needs, set directions and targets, and establish 'distance travelled' in relation to professional learning (Bishop, Clarke & Morony, 2006). One of the features of these lists is that they focus on goals beyond the systemic issues that often dominate professional development agendas.

There is clear evidence already of the systemic uptake of the AAMT Standards of Excellence in Teaching Mathematics in Australian Schools in a number of jurisdictions, including Queensland (through the Queensland College of Teachers), the Melbourne Catholic Education Office, and the Northern Territory Department of Education, Employment and Training [AAMT, 31; Queensland College of Teachers, 12; Catholic Education Victoria, 50; Northern Territory Department of Education, Employment and Training, 37].

### **Recommendation 10:**

That the Australian Association of Mathematics Teachers *Standards for Excellence in Teaching Mathematics in Australian Schools* be used as a framework for professionalism in the teaching of mathematics and inform the development of the forthcoming national numeracy teaching standards.

## **Effective professional development programmes**

Submissions received by the review acknowledge that 'one-off' professional development events do not build sustainable changes to teacher practice and can have little impact on the mathematics classroom environments [Archdiocese of Canberra and Goulburn Catholic Education Office, 44; Queensland College of Teachers, 12].

Education systems and projects are increasingly using professional development models that acknowledge teachers as active learners and that recognise that sustainable improvements to teacher practice result from classroom interaction and professional dialogue with colleagues in education settings. They also recognise that professional development that focuses on teachers understanding how children's mathematical understandings develop can be used to build teacher pedagogical content knowledge.

Professional development models and programmes that support teacher and student learning simultaneously were highlighted as successful in several submissions. These vary from small scale, single school projects to system-wide programmes such as The Western Australian Department of Education and Training's *Getting it Right* strategy [40] and the *Count Me In Too* programme in New South Wales [35].

Most of these exemplary programmes identify factors attributable to successful development of teachers and improved learning outcomes for students that include:

- a whole school approach (primary) and faculty (secondary) to a programme involving teach commitment
- school-based and focusing on the day-to-day work of teaching
- strong school leadership
- de-privatisation of practice; teachers engaging in professional dialogue about their pedagogies, assessment practices, use of data for planning, and qualities of student work
- a focus on collaborative problem solving

- programmes extended over extended period of time involving follow-up and support for further learning
- support from sources external to the school that can provide on-going input of 'what works' in other locations (including national and international)
- evidence-based programmes underpinned by research.

There is a range of professional development provision for mathematics teachers in Australia. Evaluations of these sometimes include measures of impact on student learning but more commonly the focus is on teacher impact measures. Recent initiatives such as the Australian School Innovation in Science, Technology and Mathematics (ASISTM) provide for extended projects, but to date evaluation results are not available for this initiative, which has involved around 300 projects.

The Panel was pleased to read of the many and varied professional development programmes occurring with obvious benefit across Australia.

Features which emerged as relatively common across the more successful programmes included:

- a focus on involving whole schools (or at least all teachers at the relevant grade levels)
- the development and use of research-based frameworks (e.g. *Count Me In (NSW)*, *First Steps (WA)*, *Early Numeracy Research Project (VIC)*, *Possible Learning Connections Framework* [Catholic Education South Australia, 4])
- partnerships between schools, systems, and universities
- a recognition of the importance of school leadership in effective programmes
- an assessment focus on understanding individual student thinking in relevant mathematical domains, often involving one-to-one interviews, as well as typical learning trajectories for these (recognising that such trajectories will not apply for all students)
- enhanced pedagogical content knowledge as the major focus
- a clear link to classroom practice, with opportunities for peer or other expert support within classrooms
- strategies for addressing the needs of low-attaining students
- ongoing reflective professional development.

Groves et al. (2006), in their report on primary numeracy, summarised findings from research on professional development indicating that effective programmes:

- provide teachers with the time and appropriate resources to enable them to reflect on their teaching and make changes as and when they see fit – a major impediment to change identified by teachers was a perception of a lack of time to adopt new practices
- provide continuing support and encouragement while teachers are exploring possibilities and trialling new strategies in their classrooms
- involve teachers in school-based and wider networks
- are of sufficient duration (time span and contact hours) to allow significant changes to habitual beliefs and practices
- create opportunities for the exploration of, and reflection on theory-practice relationships.

Moreover, research in both Australia and overseas has emphasised the importance of professional development being:

- content-focused
- situated in or near classrooms where teachers work
- embedded in the curriculum they teach (Groves et al., 2006).

There is a range of aspects of teaching that could productively form the basis of sustained teacher professional learning. It seems that the quality of mathematics teaching will be linked to ongoing opportunities for teachers to extend their knowledge of mathematics teaching and learning, and that well resourced programmes can assist in this.

Submissions have included many examples of well-structured and effective professional development programmes. It is clear that these have been particularly strong in the early years, with exemplary programmes including Count Me In, First Steps, and the Early Numeracy Research Project. Given the evidence in the literature and in submissions regarding the worrying quality of mathematics teaching in the middle years (see, e.g. Hollingsworth et al., 2003), the development of similar programmes, with equivalent scope and resourcing, seems essential for teachers and students in these years.

#### **Recommendation 11:**

That the research-based professional development programmes identified in this report as exemplary in supporting early and primary years' teachers to enhance numeracy outcomes be extended in their reach and impact; further that these programmes or others developed on similar principles be extended to include teachers of students up to Year 10. Exemplary professional development programmes are based on:

- enhancing pedagogical content knowledge (that is, knowledge about teaching specific mathematical content)
- providing teachers and support staff with approaches for accessing the thinking of individual students
- the premise of high expectations of all students and provide conceptually rich strategies for addressing the needs of those not achieving well
- a strong theory-practice link including partnerships between schools, systems and universities
- providing sustained opportunities for teacher learning and reflection and collegial and/or specialist support.

#### **Recommendation 12:**

That pedagogical content knowledge (that is, knowledge about teaching specific mathematical content) be a prime focus of both pre-service and in-service programmes for teachers of mathematics across all the years of schooling.

#### **Recommendation 14:**

That, in recognition of the likely continued reliance in the medium term on teachers teaching secondary mathematics 'out of field', systems develop strategies to support such teachers to improve the depth and extent of their mathematical and pedagogical content knowledge.

## 4.5 Mathematics curriculum leadership

There is also substantial evidence indicating that the quality of a school's mathematics teaching programme is also dependent on the quality of the structure and practices of the school Mathematics department as a whole (e.g. Harris, Jamieson, Russ, 1995). There is growing interest in knowing how curriculum frameworks are implemented in schools, which mechanisms are in place for in-school curriculum leadership, and how progress is being monitored (e.g. Horwood, 1998; Jacob & Frid, 1998).

However, from a research perspective, it is noticeable that there has been little attention paid to the organisation, structure, and activities of the group of teachers in any school who teach mathematics. Limited research has been focussed on these crucial aspects of schools' approaches to *mathematics* education. 'The department head structure has been the taken-for-granted means of organizing secondary schools ... and yet little is known about how this structure influences the teaching/learning or the change processes' (Hannay & Erb, 1999, pp.2–3).

An important exception to this has been AESOP, as reported in the SiMERR submission [25]. Using 50 case studies of mathematics departments for which evidence of the achievement of outstanding educational outcomes was present, the research team identified 'seven major elements in relation to the school, faculty, characteristics of teachers, pedagogical practices, and parents and students'. Major themes to emerge in these settings included a strong sense of team, qualified staff with a breadth and depth of experience, solid teaching, time on task maximised, assessment as a catalyst for teacher cohesion, a clear mission of high expectations, and catering for students in their learning.

There is an increased emphasis on the need for mathematics curriculum leadership. For example, in the Early Numeracy Research Project, an Early Years Numeracy Coordinator was a key part of the original model but was also important to the success of the project. While some of the role was administrative, there were key mentoring and coaching required (Clarke et al., 2002).

The establishment of *Professional Learning Teams* acknowledged the need for supportive in school structures to support teacher learning. Data from principals, coordinators and teachers confirmed that participation in professional learning teams stimulated growth in four main areas:

- knowledge about the teaching and learning of mathematics
- the capacity to cater for the needs of individual students
- attitudes to and personal confidence with mathematics
- the level of teamwork and collegiality (Clarke et al., 2002, p.28).
- In relation to professional learning teams, it was recommended that:
  - schools form professional learning teams that focus on mathematics education, to provide a forum for collegial discourse, professional development, and team monitoring of student performance in mathematics
  - ongoing sustained professional development in mathematics education, centrally and in regional clusters, be provided at the professional learning team level.

## Specialist teachers of mathematics in the primary school

One suggestion to improve the teaching of mathematics in primary schools is to provide specialist teachers of mathematics. Reys and Fennell (2003) make an argument for elementary school mathematics specialists in the US context. They present two models, one a lead-teacher model where the teacher is relieved from classroom responsibilities to specialise on mathematics and specific teacher support. This has some similarities to the numeracy coordinator role. The other model involves shared responsibilities across two upper grades. While the authors provide examples where these have been used and a clear justification, limited evidence is provided of the impact.

There is anecdotal evidence that the *isolated* teacher in the classroom is still a prevalent model in many schools. The implication is that individual teachers must take major responsibility for student achievement (or lack of). Evidence is emerging, however, that schools that are successful in terms of mathematics achievement have a well-determined departmental structure, with much collaboration and sharing of teaching and curriculum ideas, stimulated by an active and well supported Head of Mathematics (ACER, 2003; Clarke et al., 2002). Indeed, school-based curriculum leadership is vital in the establishment of effective processes, targeted resource provision, teacher discipline renewal, and broad-based teacher professional development. School mathematics departments are also likely to play an important role in classroom teachers' translation of the intended curriculum (specified at the national, state, or institutional level) to the implemented curriculum (see Lokan & Greenwood, 2001). These kinds of decisions require considerable planning, leadership and change management. Effective departmental leadership in British secondary schools has been shown to lead to low staff turnover, vision, a climate for change, collegiality, and sound organization, resource management and monitoring systems (Harris, Jamieson, Russ, 1995). That at least 50% of teachers of mathematics and science in Australia indicated a preference to leave teaching if given the opportunity – one of the highest percentages internationally (Lokan, Ford, & Greenwood, 1996) – implies a clear need to investigate the professional culture of school mathematics departments. The retention of teachers who have had a substantial investment of social capital in their initial training and subsequent professional development is another important strategic factor in educational provision and planning.

### **Recommendation 15:**

That structured programmes be implemented to support teachers to develop the knowledge and skills necessary to exercise effective leadership roles in numeracy and mathematics within schools.





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## *Appendix 1: Discussion Paper*

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# *Discussion Paper for the National Numeracy Review June 2007*

*This paper is intended to assist submission respondents in organising their submissions in relation to the Review's Terms of Reference.*

*It provides a guide for the ordering of information only and is not intended to be prescriptive in terms of the way submissions are structured.*



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## *A Note from the Chair*

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To assist in the development and implementation of reforms to improve literacy and numeracy, Senior Officials from Commonwealth, State and Territory governments have agreed to conduct a Review to identify the teaching, learning and assessment practices that lead to improved numeracy outcomes for students. This is to be done through an examination of the currently available evidence. This examination will provide guidance to policy makers in relation to the critical issues associated with and impacting on effective mathematics teaching and the provision of appropriately situated numeracy education in our school systems.

This Discussion Paper has been developed to guide stakeholder responses in the key component areas associated with numeracy teaching and learning. In drafting this Discussion Paper it is widely acknowledged that the societal expectations for numeracy development are significantly different today from those of the past. Changing workforce demands require a numerically literate society to sustain human capital. Understanding how numeracy teaching and learning is being played out at the school and classroom level, and how transferable those skills are to meet an individual's needs for later life is central to considerations. To this extent this Discussion Paper frames a series of emerging questions.

Numeracy teaching is a core responsibility of all teachers and school education authorities. For a decade or more education systems, researchers and individuals have been involved in a range of innovations. Many of these activities, developed to improve outcomes in numeracy have undergone a documented evaluation. The Review is not collecting opinions or views but invites provision of evaluations and research evidence which inform practice to achieve better outcomes for students. It is about bringing together the evidence for all of this work to enable us to make, on the basis of probability, a set of recommendations as to which strategies have been found to work.

I invite you to provide evidence, collected in your school, through your education system, or through other research which has informed practice. In considering the questions posed you may want to present case examples which have informed successful practice. Evidence that is corroborated at a number of levels will provide a valuable contribution.

In general, those managing and providing education to Australia's youth show a passion for learning and achievement. This review of current and significant research in the field of numeracy and mathematics will provide clear direction for teacher preparation and teaching and learning practices to further support teachers in meeting their objectives in improving learning outcomes.



**Professor Gordon Stanley**

Chair, National Numeracy Review

For the Human Capital Working Group of the Council of Australian Governments



# Information for Participants

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## Key dates

12 June 2007	Discussion Paper available
16 July 2007	Submissions due
24 July 2007	Invitational Review Forum (Melbourne) to synthesise outcomes of submissions
Early August 2007	Draft report to Human Capital Working Group (COAG)
Late August 2007	Report finalised

## Submission guidelines

Material provided in submissions will be referenced in the meta-analysis of research evaluation studies to provide the evidence base for recommendations. In such an analysis individual schools would not be identified without prior permission of the organisation concerned.

These Guidelines outline the requirements for submissions to the National Numeracy Review.

1. Submissions to the National Numeracy Review are invited from targeted stakeholders.
2. Submissions should address questions within the attached Discussion Paper which specifically refer to the Terms of Reference.
3. Submissions may be in the form of letters, documents or reports. Supporting documentation may be attached to submissions.
4. Submissions should be sent in hard copy and electronically. Electronic submissions need to be saved as an *MS Word* document or as a PDF. Electronic submissions should be provided by email to **numeracyreview@secretariat.com.au**. If this is not convenient, the submission may be provided on CD-ROM or computer disk.
5. Submissions may contain arguments, facts and recommendations for action, however, they should be framed within an evidence base.
6. All submissions must include a summary of issues addressed. This is to be no longer than half an A4 page in 12 point font.
7. Submissions must not contain any defamatory statements. Submissions which contain information which may lead to the identification of an individual person or school such that it may cause harm will be logged as a submission; however, the submission will not be made available publicly.
8. The individual with authority for submitting on behalf of an organisation must sign the submission, indicating the signatory's position, and at what level the submission was authorised. All submissions must include a contact name, phone number and postal address for verification purposes. If the submission is from an organisation, this should be clearly indicated.

9. Unless you request that your submission be treated confidentially, submissions may be made publicly available on a government website as part of the review process. In addition, you may wish to note that because the Australian Government may be required to release your submission by the operation of law, judicial or Parliamentary body or government agency, the Review Secretariat can give no undertaking that your submission will never be made publicly available.
10. If you would like your submission to be kept confidential, please indicate this clearly at the top of your document or in a covering note. If only part of your response is confidential, please put that part on a separate page(s).

The closing date for making submissions is *midday Monday 16 July 2007*.

### Contact details

Information relating to the National Numeracy Review can be obtained either by telephone

**Ph: 02 6295 8481** or **email** to [numeracyreview@secretariat.com.au](mailto:numeracyreview@secretariat.com.au)

Submissions should be forwarded to:

National Numeracy Review

PO Box 3318

Manuka ACT 2603

**Email:** [numeracyreview@secretariat.com.au](mailto:numeracyreview@secretariat.com.au) **Fax:** 02 6295 9277

## *Background – National numeracy review*

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On 14 July 2006, Council of Australian Governments (COAG) reaffirmed its 10 February 2006 commitment to progress the National Reform Agenda (NRA), including the human capital agenda. This agenda is a long-term and integrated reform agenda across governments and portfolios, with the objective of increasing the nation's productivity and workforce participation.

COAG agreed that one of the initial priority areas would be literacy and numeracy – with the aim of improving student outcomes in literacy and numeracy. Literacy and numeracy skills are strongly correlated to success in school, students staying at school to year 12 and to further education and work. Improved numeracy outcomes will encourage higher school retention rates, increase human capital and support economic prosperity.

While Australia performs significantly above the Organization for Economic Cooperation and Development (OECD) average on literacy and numeracy, overall performance is still below the world's best and the distribution of outcomes is wider than in many countries. High average performance masks large gaps in achievement and Australia has a relatively higher level of variance within schools (i.e., between individual students). Indigenous students and those from lower socio-economic backgrounds attain, on average, lower levels of achievement.

On 13 April 2007, COAG announced, as part of progress on human capital reform, measures to improve literacy and numeracy outcomes. COAG has agreed to develop a core set of nationally-consistent teacher standards for literacy and numeracy by the end of 2007, to accredit university teacher education courses and register or accredit teachers to meet these national standards by 2009, implement on entry to school diagnostic assessment systems for children in their first year of school by 2010 and develop a core set of nationally agreed skills, knowledge and attributes for school principals by the end of 2007. The 13 April 2007 Communiqué is available at: <http://www.coag.gov.au>.

To assist in the development and implementation of reforms to improve literacy and numeracy, Senior Officials from Commonwealth, State and Territory governments have agreed to conduct a numeracy review to provide advice and recommendations to COAG's Human Capital Working Group (HCWG) through identifying best practice in teaching, learning and assessment that leads to improved numeracy outcomes for students. This is to include identification of the evidence available in relation to current and significant research incorporating clear directions for the development of teacher standards to improve the teaching of numeracy. The Review will include an analysis of national and international research and an examination of research and evaluations of mathematics and numeracy teaching in Australia.

The Review has been agreed through COAG's HCWG and is being managed by the HCWG. The HCWG has agreed that the Review will involve targeted consultation with government and non-government education authorities, the teaching profession, universities, parents and researchers.

The HCWG has appointed a panel of experts, chaired by Professor Gordon Stanley, to progress the Review. Panel member biographies are at **Attachment A**.

A Reference Group has been formed with representatives from each jurisdiction. The Reference Group will assist the panel with appropriate information sharing, particularly in relation to effective practices

for improving numeracy outcomes, and providing feedback in relation to Review documents. Reference Group membership is at **Attachment B**.

The Review will synthesise information into a publicly accessible format. The Review is working towards completion by late August 2007.

The Review will:

- identify the evidence base for mathematics and numeracy teaching;
- examine the preparedness of graduates for the teaching of numeracy;
- examine the adequacy of professional development activities in mathematics and numeracy; and
- underpin the development of teacher standards for numeracy, which will in turn inform other government initiatives to improve teacher preparation and teacher professional development.

The Terms of Reference for the Review are at **Attachment C**.



# Issues and key questions

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These issues and key questions draw from the Background Paper developed for this Review (available at: [http://www.dest.gov.au/sectors/school\\_education/policy\\_initiatives\\_reviews/policy\\_initiatives\\_reviews\\_menu.htm](http://www.dest.gov.au/sectors/school_education/policy_initiatives_reviews/policy_initiatives_reviews_menu.htm) ). Full references are only included where supplementary to the Background Paper.

## 1. Numeracy and Mathematics in Australia

### What is numeracy?

Numeracy is at times thought of as a subset of mathematics, the ‘basic mathematics’ needed for every day or perhaps the basic building blocks of mathematics, and at other times as somewhat more than mathematics involving a grasp of the interplay between mathematics and the social contexts within which it is used. Clearly there are ambiguities, with the terms ‘mathematics’ and ‘numeracy’ being used almost interchangeably and at other times regarded as quite distinct. The Australian Association of Mathematics Teachers (AAMT), following Willis (1998), defined being numerate as being able to:

*‘... use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.’ (AAMT, 1998, p.1)*

Over recent years, the term, ‘mathematical literacy’ has become more widely used internationally, with the *Programme for International Student Assessment* (PISA) using the following definition of Mathematical Literacy:

*‘An individual’s capacity to identify and understand the role mathematics plays in the world, to make well-founded judgements, and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen.’ (OECD, 2004, p.15)*

### Educating for numeracy

Broad structural and cultural changes that affect education continue to occur in meeting global and local economic needs and goals. These include an economic shift towards serviced-based and knowledge-intensive industries, major changes in the kind of working lives that young people of today can expect, compared to those of their parents, and advances in information and communications technologies. However, the other roles of education - beside making useful workers - include creating fully participating citizens and shaping people’s identities. The dynamic and inter-connected nature of all of these has a significant impact on mathematics and numeracy education for now and for the future.

Numeracy education, as Jablonka (2003) argues, has different emphases and goals from mathematics. These goals include contributing to developing human capital, for cultural identity, for environmental awareness and social change, and for evaluating mathematics.

The PISA and the *Trends in Mathematics and Science Study* (TIMSS) offer useful insights in considering the development of numeracy, through their levels of achievement (in the case of PISA) and through benchmarks and content descriptors (in the case of TIMSS).

These studies and other numeracy projects suggest that the curriculum needs to emphasise the development of at least three distinct dimensions of numerate behaviour:

1. the processes, procedures, skills and strategies involved in the choice and use of mathematics
2. the mathematical knowledge to be understood and applied; and
3. the situations and contexts within which numeracy practices are experienced.

Often it is assumed that these three dimensions can be developed separately and somehow come together at the end to produce numerate behaviour. Unfortunately, this integration does not appear to occur as readily as has been assumed even amongst well achieving students.

While in some schools, students are learning to use mathematics across other subjects to participate in 'real-life' problem solving, in other schools subjects which could and should draw on mathematics, are actually 'de-mathetised' in an apparent effort to make them more accessible.

There are examples of cross curriculum approaches to numeracy in which schools participate in learning communities so that school learning in mathematics, science and technology are linked with industry. Enthusiastic teachers are enabling students to learn in contexts directly relevant to them personally and their community.

Despite recent developments in some Australian mathematics curricula which have resulted in a greater emphasis on thinking mathematically, many teachers report considerable pressure to focus on superficial learning rather than a more in depth knowledge of mathematical concepts. The aim for many teachers is still 'getting through the course.' This was reflected in the TIMSS Video Study of 638 Year 8 lessons from seven participating countries (Hollingworth, Lokan, & McCrae, 2003). One mathematics teacher, in each of 87 Australian schools, was chosen randomly and videotaped. Australian findings showed that a large proportion (>3/4) of problems were low in procedural complexity. This was the highest of any country in the study. Just over a quarter of problems used real-life connections (compared to 42% in The Netherlands) and less than 10% of problems had more than one solution.

Similarly, the organisation of mathematics curricula into strands that classify mathematics as a strictly compartmentalised discipline makes it 'almost impossible for students to see mathematics as a continuously growing scientific field that continually spreads into new fields and applications. Students are not positioned to see overarching concepts and relations, so mathematics appears to be a collection of fragmented pieces of factual knowledge.' (de Lange, Jan (2006) Mathematical literacy for living from OECD-PISA perspective Tsukuba Journal of Educational Study in Mathematics vol 25)

Hollingworth et. al. (2003) noted that, 'Australian students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking.' They further commented that, 'there is an over-emphasis on 'correct' use of the 'correct' procedure to obtain 'the' correct answer. Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on are rare' and reported 'a syndrome of shallow teaching, where students are asked to follow procedures without reasons'.

Over the past decades, studies on workplace numeracy in Australia and internationally (refer to Background Paper to Review) have suggested that the numeracy needs of an adult in the workforce are not met by the school mathematics curriculum. The findings of these research projects contrast adult and workforce needs with that of perceived intent of the school curricula:

- In school the object of activity is for students to learn mathematics in a supportive environment, whereas in the workplace the object is to achieve a productive outcome under constraints of time, money, safety, legislative requirements, etc., and mathematics is but one tool or mediating artefact in this process.
- The mathematics used in the workplace is often invisible or viewed as relatively low-level when compared to lists of school mathematics topics, but it actually requires substantial depth of understanding and mistakes are to be avoided at all costs.
- In the workplace, knowledge of context and content is of the essence. Judgements are made, often instantaneously, in the light of all available quantitative and qualitative information, including historical records and sensory data on physical conditions as well as dynamic technology-generated data.
- Among the hybrid of generic competencies required in practice, communication plays a vital role, especially in times of breakdown in equipment or understanding, and it is at these times the visibility of the mathematics can come clearly into focus.
- Knowledge and skills are not simply 'applied' but transformed with (locally) new knowledge created by adults as citizens and/or workers in response to unpredictable and ever-evolving problems. The transfer of school mathematical knowledge cannot be assumed.

This raises a number of questions for school mathematics curricula and the curriculum more broadly. Firstly, do the mathematical needs of an adult in the workforce differ from that envisaged in the design of school curriculum or are curricula appropriate but the ways in which they are taught problematic? Secondly, is it possible for schools ever to mimic the complexity involved in the application of mathematical knowledge to real tasks in real workplaces? And thirdly, it begs the question of to 'which workplaces' we are referring? Scientists and mathematicians, for example, are also in the workforce and presumably their workplace requirements also need to be met. The question is not simply whether the school mathematics curriculum, as designed and/or as taught and learned, prepares people for the workplace, but rather which workplaces (and other places) it does and does not serve well and how we address the very different needs of adult life.

### **Australia's mathematical skill base**

There are two complementary aspects to the development of Australia's mathematical skill base: levels of achievement in mathematics and levels of participation in mathematics.

With respect to levels of achievement, most Australian school students currently participate in state and territory assessments of numeracy in Years 3, 5 and 7 and, in some states and territories, also in Year 9. The benchmarks are considered to describe a minimum standard without which students will have difficulty making progress at school. A significant proportion of Australian students are not achieving these benchmarks. In 2005, the percentage of students not meeting benchmarks in numeracy was 6% of year 3 students rising to 9% of students by Year 5, and 18% by year 7).

The TIMSS and OECD's PISA Programme are the two most widely cited international comparative assessments. It is generally considered that the primary focus of TIMSS is mathematics, as such, while the primary focus of PISA is mathematical literacy as described earlier, which in the Australian context is thought of as numeracy. Countries do not necessarily achieve at similar levels on each of these assessments - some countries do very well on TIMSS but not so well on PISA and for others the reverse is true. This suggests that they assess rather different things.

PISA measures the performance of fifteen-year-old students in over 30 OECD nations, and remains our best international guide to performance. In 2003, as part of PISA testing programme, 12,500 fifteen-year-old students, from all schools systems, and from each state and territory, completed a two-hour pen and paper numeracy test in their schools, and answered a 30 minute questionnaire. The focus of the assessment was on how well young people had been prepared to meet challenges, how well they could adapt their learning to the needs of their lives, and to address aspects of school organisation, including factors contributing to disadvantage.

The results of 2003 PISA show that Australian students perform well in numeracy and in problem solving overall; with only four countries outperforming the group of eleven similarly performing countries that included Australia. In numeracy, Australia's results are strong, being above the OECD average in problem solving and each of the mathematical literacy subscales (OECD 2004). Similarly, results of the Third TIMSS conducted throughout the 1990s shows that Australian students consistently perform above the international average. As a generalisation, Australia does better on PISA than TIMSS.

While Australia did well on the PISA and the TIMSS assessment (of 1994), there is greater disparity between high achieving and underperforming students than in most other countries. Numeracy policy development thus needs to focus on the significant gap between high achieving and underperforming students, to improve the outcomes for underperforming students while concurrently ensuring that Australia's performance remains high.

With respect to levels of participation in mathematics, industry, business and the higher education sector in Australia have flagged an emerging shortage of qualified mathematicians and statisticians. *The Australian Council of Deans of Science Report* (2006) and the *National Strategic Review of Mathematical Sciences Research in Australia* (AAS, 2006) each urge a greater emphasis on the preparation of mathematicians, such preparation involving all levels of education.

Recent media attention has focused on the shortage of a suitably qualified teaching workforce in mathematics. We need to attract more people into mathematics teaching and also to establish the skills required by teachers if they are to engage students fully in mathematics, resulting both in a mathematically literate population and a new generation of students choosing to study mathematics beyond school.

### To consider:

1. Consider the dual goals of numeracy and mathematics, i.e., those of developing a broadly numerate workforce/community and the next generation of highly skilled people for the mathematically oriented professions:
  - To what extent are these goals compatible?
  - To what extent can/should they be addressed simultaneously in curriculum documents and by classroom teachers?
  - What should the balance be between these goals/foci?
  - Do these goals remain parallel during schooling or do they diverge at some point?
2. How well do school curriculum and pedagogy support relationships between numeracy and mathematics, science and other subject areas?

## 2. School organisation and supporting structures

Across and within jurisdictions and school sectors, organisational groupings are commonly established around the early, middle and upper school years. For the purpose of this document the following applies:

- early years of schooling (typically K to years 3/4);
- middle years of schooling (typically years 5 to 8/9); and
- upper years of schooling (typically years 9/10 to year 12 or equivalent).

In general, where we ask questions about schooling, unless we specify early, middle or upper, the reader can assume that the question relates to all three.

### What institutional practices are effective for numeracy education?

Various attempts have been made to define those characteristics that produce effective numeracy outcomes within schools. Four such practices that will be addressed here include: structural arrangements; ability groupings; differentiated curricula; and the use of Information and Communication Technologies (ICT). Characteristics of effective teachers will be addressed in Section 3.

Organisational arrangements in schools, including structural arrangements and human relations policies, can either inhibit or enhance whole of school models for numeracy education. Opportunities may exist where small teaching teams across curriculum work with students addressing key numeracy concepts in different areas of the curriculum.

Structural arrangements within the mathematics classroom are also relevant. How classroom organisational structures impact on challenging students of all abilities to maximise potential raises questions for inquiry. The application of ability level groupings as an alternative to mixed class abilities occurs in some schools. Whether or not this improves numeracy outcomes or rather is a practical response to other demands is unclear.

Many countries have differentiated curricula, that is, different curricula designed for students regarded as being of different abilities of achievements. So too do many Australian schools. Margaret Brown's review of research in primary schooling around the world noted that there was a negative correlation between

a country's overall performance and the extent to which they differentiated the curriculum for different perceived abilities, indicating that overall performance goes down when the level of differentiation between curriculum goes up (Brown et al, 1998). There is a range of ways of differentiation, in seeking to meet the needs of all learners, including differentiating by quantity, by task, by level of support, and by outcome. An examination of the drivers for differentiated curriculum in Australia and the corresponding impacts on student outcomes is worth consideration.

## Information and Communication Technologies (ICT)

In terms of supporting structures that contribute to numeracy and mathematics outcomes, the effective use of ICT needs examination. For example, computational calculators and computers, used appropriately, have the potential to produce gains in numeracy outcomes in classrooms as well as the capacity to bring about rich mathematical understandings. The use of computational technologies poses opportunities to remove the focus on tedious drill and practice of skills to higher-order decision making and interpretation.

Although there are relatively few studies in this area, research suggests that while there is use of computers in classrooms, the computers are used for low level tasks. The uptake of ICT for higher-level tasks has been limited in mathematics classrooms in Australian schools.

Barriers to the use of ICT in the classroom result from both human and physical infrastructural impediments. In many schools, simple lack of access to computers or appropriate software is an issue. While in others, the skills, confidence and beliefs of teachers in relation to the use of ICT places limitations on students' opportunities. The use of ICT in pre-service education as well as teacher professional development requires consideration both at the technical level and also at a confidence level to overcome these barriers to use of ICT.

Many teachers are making use of a subset of computer based skills as most of today's students are familiar with computers and the internet. This provides an engaging environment as students in Australia make 'connections' with students and experts all over the world. However, the extent to which higher order challenges are posed for students through ICT remains questionable. What needs to be considered is whether engaging in studies through computational technologies such as graphics calculators improves numeracy outcomes. There is a shortage of quality research on the issue of engagement through ICT and its impact on learning (see, for example, [www.becta.org.uk](http://www.becta.org.uk)).

### To consider:

#### School organisation and supporting structures

3. Based on evidence, what structural arrangements in schools are found to be more conducive to improving numeracy outcomes?
4. What evidence can you provide of the depth of knowledge and understanding of numeracy and mathematics needed for primary teachers to be able to teach in primary schools across the grade levels?

#### Information and Communication Technologies (ICT)

5. Where ICT programmes have been evaluated in regard to mathematics and numeracy outcomes, provide evidence of how they either enhanced or did not enhance outcomes

### 3. Effective teaching practices

A number of studies indicate that effective teachers tend to be those who:

- had 'connectionist' orientations (as opposed to 'transmission' or 'discovery' orientations);
- focused on students' mathematical learning (rather than on provision of pleasant classroom experiences);
- provided a challenging curriculum (rather than a comforting experience); and
- held high expectations of initially low-attaining students.

Many of these features constitute meaningful and constructive classroom interactions between teacher and students, and perhaps also amongst students. Indeed, students connecting with the subject and with peers and teachers is one of the characteristics identified as making the difference to numeracy attainment within classrooms in a large-scale, 65-school numeracy study in New South Wales (Busatto, 2001).

Literacy and numeracy is foundational to success in schooling. The National Literacy and Numeracy Plan recognises the importance of both literacy and numeracy as the cornerstones of education. Yet, the inter-relationship between the two domains is often neglected despite the implications of language and literacy issues associated with the learning of mathematics. A few pertinent research findings illustrate this relationship.

At the upper primary level, Newman (1977) examined the errors made by students as they solved worded mathematics problems, finding that at least 35% of the errors made occurred before students were even able to attempt to apply mathematical skills or knowledge. These language based errors occurred during the reading, comprehension, and transformation stages. Later research by Clements (1980) and Clarkson (1983) confirmed Newman's findings.

The context in which a mathematical problem is set has the potential to disadvantage those who are unfamiliar with it. Zevenbergen (2001) contended that children's familiarity with aspects of language is related to their socio-economic backgrounds and this could also affect mathematical performance. Doyle (2005) maintained that literacy, with respect to the ability to read a given text, was an essential part of the mathematical problem solving process.

While understanding the language through which mathematics is taught may be important for the avoidance of language specific errors, it is not sufficient in itself. Understanding of mathematical language is also a necessity to being numerically and mathematically literate. Transferring such concepts to students of varying cultural background requires an understanding of learning styles within the cultural operating framework in order to make the appropriate linkages at the necessary learning points.

#### **How can numeracy outcomes for students of different groups be improved?**

Understanding student cohorts and the uniqueness of individuals and teachers is central to establishing appropriate practices to meet the dynamic needs of today's classrooms. Addressing the needs of those children having significant difficulty in acquiring numeracy skills is essential. Teachers' beliefs in relation to the best approach to facilitate learning impact on the approaches they use.



In respect to underperforming students or students 'at risk', there are commonly accepted approaches to support these students. Withdrawal programmes or 'in classroom' support are two approaches, however there may be others. Withdrawal programmes provide opportunities for differentiated curricula for 'at risk' students and are generally underpinned by direct instruction and explicit teaching of number facts. This approach and its effectiveness in developing the deep mathematical understanding required for sustained improvements in performance needs to be considered in the context of its resource intensity and possible alternative available options.

Programmes supporting 'at risk' students in mainstream classrooms aim to support teachers in building an inclusive classroom community. This support may be in the form of specialist teachers or mathematics coaches working with classroom teachers and support the planning and implementation of effective teaching and learning programmes.

Effective teachers know their students. As the AAMT Teacher Standards (2006) list under professional knowledge, 'excellent teachers of mathematics have a thorough knowledge of students' social and cultural contexts, the mathematics they know and use, their preferred ways of learning, and how confident they feel about learning mathematics.'

Improving the numeracy outcomes for Indigenous students is perhaps the major equity challenge facing numeracy policy in Australia. Although policy initiatives and programmes in the past have had limited success, qualities of an effective learning environment for Indigenous students are being identified through research. Successful outcomes for Indigenous students occur when teachers, for example, acknowledge and accommodate socio-cultural differences and differences in home background, recognise the individuality of students, and value Aboriginal ways of teaching and learning (Erebus International, 2007).

Indigenous students' mathematics learning may be enhanced by accounting more for the unique learning styles of such students in their sociocultural context. Where teachers fail to recognise differences in learning styles and the cultural 'ways of knowing' that students from Indigenous backgrounds bring to the classrooms, a deficit approach to teaching Indigenous students frequently results. This can result in low teacher expectations of student achievement which are often realised (including through a 'dumbing-down' of the curriculum) and perpetuate inequitable learning outcomes.

More recent projects have benefited from lessons learnt and subsequently reported positive and promising outcomes through actively involving the Indigenous community in the development of the school mathematics curriculum. The 2006 *Mathematics in Indigenous Contexts (K-2)* project in NSW, for example, was a pilot study in which primary schools and their immediate communities were supported in developing mathematics learning activities which allowed Aboriginal students in their respective schools to demonstrate their numeracy understanding. Generally, participating students in each of the project schools improved on their pre-test scores on the NSW Schedule for Early Number Assessment [SENA], and significantly, made greater leaps in this test over the same period when compared to non-Indigenous students (Erebus International, 2007).

There is evidence that supporting teachers in their professional practice with Indigenous students can yield positive results. An issue to consider is that of teachers' beliefs in relation to the best approach to facilitate learning, not only for Indigenous students but for those with significant difficulty with developing mathematics literacy. In this context, a belief process that perpetuates teachers' notions of innate abilities

could be problematic. An examination of professional practices supportive of belief in students' capacities for learning may be central. This relates to teachers' beliefs in their students' abilities and potential and its consequential effect on student performance.

Student motivation, attitudes and performance are strongly influenced by teachers with their instructional practices including lesson goals, expectations and assumptions regarding their students' capacity impacting on learning outcomes. The value of rich assessment regimes undertaken in a manner that enables students to demonstrate their learning, which may not be confined to the written testing environment may form part of a teacher's repertoire of skills to enhance numeracy learning outcomes.

In relation to the use of aids in teaching, many teachers use concrete materials to facilitate learners to reach the answer. Teachers must also be cognisant of the use of concrete materials to gain a deeper understanding of the concepts. There are multiple forms of learning aids which can cause cognitive confusion rather than promoting connections. In considering classroom practice, the use of aids should consolidate rather than confuse.

Research shows the importance of students 'connecting' with the subject, with their peers and with their teachers to achieve numeracy outcomes. Effective teachers provide focused, developmentally appropriate and engaging activities for their students.

What are Australian students looking for in a learning environment? A Dusseldorf Skills Forum Report of October 2006 investigated the *Views of Gen Y* Australians (16-24 year olds) by way of a qualitative study. This report found that 'young people had a strictly instrumentalist view of education', viewing education as a way of providing the skills and knowledge necessary to get a job. (Saulwick Muller Social Research, 2006).

### **To consider:**

#### **Teaching strategies for different groups**

- 6.** To what extent are differences in numeracy and mathematics achievements observed amongst different groups due to general issues of disadvantage and to what extent are they specific to numeracy and mathematics?
- 7.** What strategies/programmes are effective to assist students who are at risk in aspects of numeracy learning including students with physical and or learning difficulties? Please provide evidence.
- 8.** What is the impact of students' attitudes to numeracy and mathematics on their life choices?
- 9.** What evidence do you have of successful attempts to address the issue of student motivation in numeracy and mathematics, particularly in relation to the teacher's expectations and assumptions regarding their students' capacity? (Please include case studies).
- 10.** What evidence do you have of programmes that are successful at enhancing the numeracy and mathematics learning of Indigenous Students?
- 11.** Are you familiar with any programmes focused on reducing anxiety levels towards numeracy and mathematics on the part of teachers and students?
- 12.** How do students develop best the capacity to make appropriate choices between, and effectively implement, mental computation, written methods and the use of technologies?

## Classroom numeracy assessment

The most important assessment opportunities for improving student learning are those used by teachers to improve teaching and learning, and the most important opportunities are those that are formative. This is widely recognized as a key component of learning, and there is a multitude of sets of advice for teachers, both written by teacher educators and by departments of education.

Interest in forms of assessment other than topic tests was first stimulated by Clarke (1988) who proposed a range of alternate assessment techniques, ranging from student self-assessments to portfolios to observational checklists.

More recently researchers have looked systematically at the role assessment could play in enhancing student learning instead of just measuring it. Gipps and Stobart (1997), for example, encapsulated this as the difference between assessment for learning and assessment of learning. The definition given by William, Lee, Harrison and Black (2004) clarified the difference:

Assessment for learning is any assessment for which the first priority in its design is to serve the purpose of promoting pupils' learning. It thus differs from assessment designed primarily to serve the purposes of accountability, or of ranking, or of certifying competence. An assessment activity can help learning if it provides information to be used as feedback, by teachers, and by their pupils, in assessing themselves and each other, to modify the teaching and learning activities in which they are engaged. (p.10)

Reviews of research have found that assessment practices can have an impact on students' attitudes and achievement (e.g. Natriello, 1987; Crooks, 1988) in both positive and negative ways. Further reviews of research (e.g. Bangert-Drowns, Kulik & Kulik, 1991; Kluger & DeNisi, 1996; Black & William, 1998) have explained in what circumstances assessment helps and when it hinders students' learning.

### To consider:

#### Assessment

13. How do existing state/territory/national assessment documents/processes inform teaching practices at the classroom level?
14. Provide evidence of the range of assessment approaches used by classroom teachers and how these are used to inform teaching practice?

## 4. Teacher education and professional development

Clarity is sought around what is expected from those teachers charged with the task of educating students to become mathematically literate members of society. Achievements and successes are important to ensure the development of all students and so it is necessary to understand the levels of specialist knowledge required of teachers educated in Australia's higher education system and supported through professional development programmes at all levels in all school systems.

A fundamental question being asked is how the preparation of teachers today and the skills imparted through pre-service education and professional development programmes are being carried forward within a dynamic context. Significant factors increasingly influencing school numeracy/mathematics and classroom environments include:

- rapid changes in the nature and accessibility of technologies, including information and communication technologies and the impact that computational technologies are having on the discipline of mathematics;
- globalisation and multiculturalism;
- the nature of knowledge and the knowledge economy (particularly the skills young people need to work with the knowledge);
- changing life experiences for students;
- changes in community expectations of schools;
- changes in employer's demands of employees;
- increases in the quality and quantity of research about learning including how students learn and the particular needs of young, middle and senior learners; and
- increases in the quality and quantity of research about how children learn mathematics.

This dynamic context has significant affect on what teachers can effectively be taught in their pre-service training and in maintaining relevancy once in the classroom.

Higher education providers and researchers are contributing to building the capacity of the teacher workforce—to ensure both content and pedagogy meet the current and emerging expectations of the community. Like many professions, increased accountability is pushing the drive for clarity in education and professional development. The recent COAG agreement to develop a core set of nationally-consistent teacher standards for literacy and numeracy, and the requirement for accreditation of university teacher education courses and teacher registration/accreditation to meet these national standards underpin development in these areas.

An understanding of the specialist knowledge necessary for teaching in the early, middle and upper years of schooling will be essential to establishing such standards and to formulate requirements of organisations and individuals to meet accreditation requirements. Consideration of the repertoire of skills for assessment for underperformers, high achievers and maximising all students' potential is expected. The AAMT Standards provide a starting point in deliberations on necessary specialist knowledge.

While there is considerable discussion about the need to assess pre-service teachers' mathematical knowledge there seems to be little agreement on what to measure and how. Pressing questions 'such as the balance of knowledge of content and knowledge of pedagogy, the nature of content knowledge useful for teaching, and the 'content' of pedagogical knowledge – have not been answered' (Hill et al., 2007, p.149). Hill et al. in their recent review of research suggest the following:

- Measure mathematical knowledge for teaching – valid teacher assessment should not be remote from what teachers do in the classroom.
- Measure with care – recognizing the advantages and disadvantages of different assessment formats.
- Use multiple approaches – to enable comprehensive appraisals.
- Meet professional standards of rigor in assessment – including validation of the results in terms of impact on students.
- Learn from other measurement methods – more cross-over needs to occur between quantitative and qualitative researchers.
- Attend to issues of equity.

- Investigate the relationship among mathematical knowledge for teaching, other domains of teaching knowledge, and student learning.
- Increase professional role and control.

Primary school mathematics teachers in particular need to have confidence in their own capacity as teachers of mathematics if they are to overcome anxieties surrounding teaching of mathematics. To build this confidence requires a considerable body of knowledge, both content and pedagogical.

The three-year Victorian Early Numeracy Research Project (Clarke et al., 2002) involved 70 schools and over 11,000 students at K-2. Provision of a research-based framework of 'growth points' in young children's mathematical learning and a task-based one-to-one assessment interview embedded in an extensive professional development programme, impacted on both teachers' background knowledge of mathematics and their confidence in teaching mathematics. Data from principals, coordinators and teachers confirmed that participation in professional learning teams stimulated growth in areas including knowledge about the teaching and learning of mathematics; teachers' capacity to cater for the needs of individual students; attitudes to, and personal confidence with mathematics; and the level of teamwork and collegiality.

A National Research Council Committee on Science and Mathematics Teacher Preparation (2001) report provided recommendations on the preparation of mathematics and science teachers to:

- develop and conduct collaborative endeavours with mathematics, education faculty, and practising K – 12 teachers with assistance from members of professional organizations and mathematics-rich businesses and industries;
- help prospective teachers to know well, understand deeply, and use effectively and creatively the fundamental content and concepts of the discipline that they will teach;
- unify, coordinate, and connect content courses in mathematics with methods courses and field experiences;
- teach content through the perspective of methods on inquiry and problem solving;
- present content in ways that allow student to appreciate the applications of mathematics;
- provide learning experiences in which mathematics is relate to and integrated with students' interests, community concerns, and societal issues;
- integrate education theory with actual teaching practice, and knowledge from mathematics teaching experience with research on how people learn mathematics;
- provide opportunities for prospective teachers to learn about and practice teaching in variety of school contexts and with diverse groups of children;
- encourage reflective inquiry into teaching through individual and collaborative study, discussion, assessment, analysis, [and] classroom-based research and practice; and
- welcome students into the professional community of educators and promote a professional vision of teaching, (cited in Sowder, 2007, p.200).

This poses a model of connected and integrated learning which stresses the importance of the mathematics content knowledge being connected to pedagogical content knowledge.

The AAMT (2006) has developed a set of 'Standards for Excellence' in teaching mathematics in Australian schools. These standards provide targets to which all mathematics teachers 'can aspire and work towards

in their professional development, based on the domain areas of professional knowledge, professional attributes and professional practice for numeracy teachers.'

### **To consider:**

#### **Teachers and professional development**

- 15.** We have a range of research describing the practices and characteristics of effective teachers of numeracy and mathematics in the primary years of schooling. What does the evidence indicate in relation to effective practices and characteristics for the secondary years of schooling?
- 16.** Is there evidence (formal evaluations) highlighting characteristic features of successful professional development programmes, either for teachers of numeracy and mathematics or for all classroom teachers, to enhance numeracy outcomes for those students not reaching their potential?
- 17.** What professional development and pre-service support should be offered to teachers of other curriculum areas, to ensure they are operating to the agreed numeracy teaching framework standards?

### **To consider:**

#### **Teacher education (pre and in-service)**

- 18.** What specialised numeracy and mathematical knowledge, including content and pedagogical is necessary for teachers of early, middle and upper levels of numeracy and mathematics teaching?
- 19.** Do numeracy and mathematics teachers enter the profession with sufficient numeracy and mathematics content and pedagogical knowledge to achieve improvements in students' numeracy learning? If not what would you recommend and why?
- 20.** What are the barriers and enablers to effective teacher education courses to improve the numeracy outcomes of students?
- 21.** To what extent are the Australian Association of Mathematics Teachers standards useful? Have any jurisdictions trialled these standards for teaching of mathematics and if so what impact do they have?





# Attachment A

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## National Numeracy Review – Biographies

**Professor Gordon Stanley (Chair)** has been President of the NSW Board of Studies since 1998 and is Honorary Professor of Education in the Faculty of Education and Social Work at the University of Sydney. Professor Stanley is also a member of the NSW Board of Vocational Education and Training Accreditation, and the Hong Kong Council for Academic Accreditation and the Board of the National Elicos Accreditation Scheme (NEAS). Professor Stanley is a Fellow of the Australian College of Educators and the Australian Psychological Society. He received the Wyndham Medal in 2004 for contributions to education in NSW and nationally, particularly in the area of Special Education and through leadership of the NSW Board of Studies during the period of restructuring of the Higher School Certificate.

Before these appointments Professor Stanley was chair of the Australian Higher Education Council and Deputy Chair of the National Board of Employment, Education and Training. He is Professor Emeritus from the University of Melbourne.

**Professor George Cooney** has been Chair in Education for 11 years at Macquarie University, before which he worked in Psychology and Mathematical Statistics. For six of the past 11 years Professor Cooney was also Director of the Teacher Education Programme with responsibility for the pre-service programmes at Macquarie. Educational measurement has been a constant research interest throughout his academic career, which has resulted in a longstanding involvement with the NSW Board of Studies.

His interests include teaching and pedagogy and the development of partnerships between universities and schools which encourage best practice in teachers and high achievement in students. Professor Cooney is a NSW Vice-Chancellors Committee representative on accreditation panels for the Higher Education Board. In 2006 he completed a major review of statewide assessments in the context of national developments for the NSW Minister of Education and Training.

**Dr Thelma Perso** is currently President, Australian Association of Mathematics Teachers (AAMT) and Executive Director, Curriculum at Education Queensland. Dr Perso taught in Western Australian secondary schools for 20 years, including eight of those as Head of Mathematics, and was the Senior Curriculum Officer for Mathematics in the Education Department of WA for six years prior to taking up positions as Manager of Curriculum Initiatives and Curriculum Renewal with the ACT Department of Education and Training. Her PhD focussed on student misconceptions in algebra and she has published in the area of improving Indigenous numeracy.

**Professor Sue Willis** is President of the Australian Council of Deans of Education (ACDE) and Dean of the Faculty of Education at Monash University. Her interests include numeracy development, professional judgment, and social justice and education. She has had extensive experience at state and national levels in curriculum work focussed particularly on mathematics and numeracy and led the research and development team for the Education Department of Western Australia's First Steps in Mathematics Programme.

**Professor Doug Clarke** is a Professor of Mathematics Education at the Australian Catholic University (Melbourne), where he directs the Mathematics Teaching and Learning Centre. From 1999 to 2002, Professor Clarke was Director of the Early Numeracy Research Project, exploring effective approaches to numeracy learning in the early years in 70 Victorian primary schools. Professor Clarke was a finalist in the 1997 Prime Minister's Awards for University Teaching and a State Nomination for the Federal Education Minister Awards for Outstanding Contribution to Improving Literacy and/or Numeracy 2003. He is currently directing the Critical Friends' component of the Australian School Innovation in Science, Technology and Mathematics Project (ASISTM).



# Attachment B

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## Reference Group Members

State/ Territory	Name
Australian Capital Territory	Ms Trish Wilks Director, Curriculum Support and Professional Learning Department of Education and Training ACT
New South Wales	Ms Gillian Shadwick General Manager, Learning and Development Department of Education and Training NSW
Northern Territory	Ms Debbie Efthymiades General Manager, Teaching, Learning and Standards Division Department of Employment, Education and Training NT
Queensland	Mr John Boustead Manager, Curriculum Department of Education, Training and the Arts QLD
South Australia	Ms Jen Emery Curriculum Superintendent, Innovation and Research Department of Education and Children's Services SA
Tasmania	Ms Denise Neal Statewide Numeracy Coordinator Department of Education TAS
Victoria	Ms Dianne Peck Acting General Manager, Student Learning Programs Division Department of Education and Training VIC
Western Australia	Ms Glenys Reid Principal Curriculum Officer Numeracy/Mathematics K-10 Department of Education and Training WA



# Attachment C

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## National Numeracy Review Terms of Reference

The National Numeracy Review will:

1. Review and analyse recent national and international research about teaching, learning and assessment practices in mathematics and other areas of learning that contribute to numeracy outcomes, including:
  - a. the relationship between numeracy and mathematics, science and other subject areas;
  - b. the dynamic and evolving nature of numeracy and of mathematics in society;
  - c. practices that are shown to be effective in improving numeracy outcomes for learners (including disadvantaged learners) and underperforming students;
  - d. practices that are shown to be effective in improving outcomes for Indigenous students; and,
  - e. evidence on the effectiveness of existing programmes, policies and projects both in Australia and internationally.
2. Identify the extent to which prospective and in-service teachers develop mathematics pedagogic content knowledge.
3. Identify the relationship between teachers' knowledge of, and confidence with, mathematics, their mathematics pedagogic content knowledge and their practice.
4. Identify effective assessment methods being used in Australia and overseas to ascertain, monitor and progress students' numeracy outcomes.
5. Produce a report of the Review's findings by **August 2007** and offer advice in effective teaching, learning and assessment practices in mathematics, and other areas of learning that contribute to numeracy outcomes, at the classroom level and in the training and on-going professional learning of teachers, based on these findings.



## *Appendix 2: Systematic reviews and best evidence synthesis*

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### **Systematic Reviews & Best Evidence Synthesis**

#### **Meta-analyses**

#### **System Reviews by Sector**

#### **ICT & CAI meta-analyses**

#### **At risk & special needs meta-analyses**



## Systematic Reviews & Best Evidence Synthesis

Authors	Year	Country Of Publication	Title	Scope	Papers Reviewed	Findings	No. & Strength Of Supporting Studies
Kyriacou, Golding & the Mathematics Education Review Group	2006	United Kingdom	Mathematics education: A systematic review of strategies to raise pupils' motivational effort in Key Stage 4 mathematics.	Systematic Review: Comprehensive review of effectiveness of interventions/strategies to raise pupil motivation in maths in years 7 to 10. Four areas identified: i) grouping ii) pupil identity iii) teaching for engagement, and iv) innovative methods.	25 studies of mixed type and quality. Of this 16 were 'evaluation' studies (none randomised, 13 naturally occurring evaluations) and 8 were correlational studies. Overall only one study was rated as high quality, 8 considered medium quality and 16 were rated low quality.	<p>1. Grouping by ability and gender were not consistently associated with any effect upon motivation. Student awareness of ability grouping was clear, with lower sets aware that their grades were limited and thus it was difficult to sustain motivation.</p> <p>2. Raising student motivational effort through developing more positive student mathematical identity uses strategies characterised by i) caring and supportive classroom environment, ii) activities which students find enjoyable and challenging, iii) focus on the development of deep mathematical understanding, iv) opportunities for student to collaborate, and v) making students feel equally valued.</p> <p>3. Teaching for engagement is characterised by similar themes to those listed for developing student identity, however, greater emphasis is on teacher care and support and making maths enjoyable.</p> <p>4. Innovative methods focusing on ICT use have a powerful impact upon student motivation. Two stages of impact are identified i) a novelty effect, and ii) an effect through development of deeper learning. Longitudinal impact requires a focus on the acceleration programmes (e.g. CAME), self-regulation strategies and innovative formative assessment can play a part in raising motivation.</p>	5 studies, unknown quality  3 studies, unknown quality
Slavin & Lake	2007	USA	Effective programmes in elementary mathematics: a best evidence synthesis.	Best Evidence Synthesis of elementary (K-5) maths outcomes in relation to all types of maths programmes available today: i) maths curricula (textbook based) ii) supplementary CAI iii) instructional process or classroom practice programmes.	87 experimental studies (randomised (36) or matched-control (51)) looking at outcomes measures not inherent to the experimental treatment over >12 weeks.	<p>1. Limited evidence of differential effects of curricula in text books - median effect size (ES=+.10). Strongest effect found with Everyday mathematics (ES=+.34) in schools using it for &gt;4yrs.</p> <p>2. Moderate effects of CAI - most studies find positive effect, especially on maths computation. Median ES=+.19, strong considering relatively rigorous studies and limited intervention time (usually 30min/week). CAI seen as good supplementary strategy.</p> <p>3. Strong positive effect from instructional processes with median ES=+.33. Of 7 processes studied; cooperative learning (e.g. Classroom Peer Tutoring, ES=+.33) &amp; professional development programmes in classroom management and motivation programmes (e.g. Missouri Mathematics Programme, ES=+.29) were most effective.</p>	13 studies, only 2 randomised. Considered marginal methodological quality  38 studies, 15 randomised. Considered moderate quality  36 studies, 19 randomised. Considered high quality

Country Of Publication			Papers Reviewed		No. & Strength Of Supporting Studies	
Authors	Year	Title	Scope	Findings		
Slavin, Lake & Groff	2007	USA Effective programmes in middle and high school mathematics: a best evidence synthesis.	Best Evidence Synthesis of : Examined middle and high school maths outcomes in relation to all types of maths programmes available today: i) maths curricula (textbook based) ii) supplementary CAI iii) instructional process or classroom practice programmes.	97 experimental studies (randomised (25) or matched-control (72)) looking at outcomes measures not inherent to the experimental treatment over >12 weeks.	1. As with elementary maths review, there was limited evidence of differential effects of text books -small median effect size (ES=+.05). Strongest effect found with Everyday mathematics (ES=+.34) in schools using it for >4yrs. 2. Moderate effects of CAI - most studies find +ve effect, especially on maths computation. Median ES=+.18, strong considering relatively rigorous studies and limited intervention time (usually 30min/week). No programme stood out as having more notable effects and effect for 'core', 'supplementary' and 'computer-managed learning' approaches were equivalent. These results were very similar to those from the elementary maths review and CAI is seen as a good strategy.	38 studies, 8 randomised. Considered mixed quality.  37 studies, 8 randomised. Considered moderate to strong quality.
					3. Strong positive effect from instructional processes with median ES=+.21. Of 6 processes studied outcomes varied with cooperative learning (median ES=+.32 in 8 studies) proving the most effective. Programmes such as STAD and IMPROVE showed very strong effects in randomised trials (ES=+.50).	22 studies, 5 randomised. Considered moderate to strong quality
The EPPI mathematics Education Review Group	2004	United Kingdom A systematic review of the impact of the daily mathematics lesson in enhancing pupil confidence and competence in early mathematics.	Systematic Review: Specific review of research relating to introduction of the UK National numeracy Strategy's daily maths lesson in K-3 ( a 45-60 minute, 3 part lesson with emphasis on whole-class teaching).	18 high quality studies identified for in-depth analysis from an initial trawl which produced 743.	1. Key features of the daily maths lesson were well received by teachers and widely implemented. However, the intention that whole-class teaching be interactive, was not realised and there is also some evidence that some implemented approaches were more 'traditional' and undermined development of a more strategic and reflective approach by teachers and may have disadvantaged lower attaining pupils. 2. Some evidence of enhanced pupil confidence and competence in early maths. However, there is also some evidence that the stricter time management posed particular problems for lower attaining students and the overall gains in student competence may have been due to improved alignment between the lesson and maths assessment, rather than broader gains in student understanding.	18 studies, classified as evaluation or description studies. Considered marginal quality. Tended to explore relationships rather than examine causality.

Country Of Publication		Title		Scope		Papers Reviewed		Findings		No. & Strength Of Supporting Studies	
Authors	Year	Country Of Publication	Title	Scope	Scope	Papers Reviewed	Findings	No. & Strength Of Supporting Studies			
Torgerson, Porthouse, & Brooks	2005	United Kingdom	A Systematic review of controlled trials evaluating interventions in adult literacy and numeracy.	Systematic Review: Examined adult education in literacy and numeracy published between 1980 and 2002. Papers included in the review had to be controlled trials evaluating interventions aimed at increasing the literacy and numeracy in study populations of adults.	A total of 4,555 potential relevant papers were identified and after various exclusions 59 papers were coded. Of these 27 controlled trials which evaluated strategies and pedagogies designed to increase adult literacy and numeracy were included. Of these nine control trials were examined for this paper, 3 examined numeracy outcomes.	1. Inconclusive: In this systematic review nine literacy and numeracy interventions for adult control trials were able to calculate the effect sizes from the data included in the studies. There have been few attempts to expose common adult literacy or numeracy programmes to rigorous evaluation. The review does provide a strong steer for the direction to be taken by educational researchers. Because of the present inadequate evidence base rigorously designed randomised controlled trials and quasi-experiments are required.	18 controlled trials without effect sizes, 9 control trials complete data, no randomised studies				
Anthony & Walshaw	2007	New Zealand	Effective pedagogy in Mathematics/ Pangarau.	Best Evidence Synthesis: Aims to 1. identify and explain the characteristics of pedagogical approaches that enhance Mathematics/ Pangarau proficiency in NZ. 2. identify pedagogical approaches that make a significant difference for diverse learners in early childhood and school years in Mathematics/ Pangarau in NZ.	total number of sources cited was over 1,100. A total of 660 papers are referred to in the report. Including non-empirical discussion documents. Evidence spanned sectors: preschools (18%), primary(48%), secondary (21%), teacher education (6%).	1. Sound teacher subject matter knowledge and pedagogical content knowledge are required to assess student understanding and for adapting tasks, activities and resources to build students' mathematical knowledge. Effective teachers use their subject matter knowledge to make decisions on lesson content that provide opportunities for students to develop their mathematical understandings and their cultural identity. 2. Building home-community and school-centre partnerships has a positive effect. 3. Early childhood research proposes that a balance between teacher initiated numeracy group work and student initiated instructive numeracy play activities is beneficial. Within childcare centres and schools effective teachers create opportunities for students to develop their mathematical and cultural identities. 4. There is conclusive evidence that effective mathematics teaching bridges the gap between learners' intuitive understanding and the conventional mathematics understandings sanctioned by the world at large.	Research quality not commented on. Evidence drawn from the full range of research methodologies. No effect sizes given.				

## Meta-analyses

Authors	Year	Title	Types Of Studies Reviewed	No. Of Supporting Studies	Findings	Relevant Combined Effect Sizes	Comments
Ahn & Choi	2004	Teachers' subject matter knowledge as a teacher qualification: A synthesis of the quantitative literature on students' mathematics achievement.	Studies (1964-2004) of various types, examining teacher qualifications/ knowledge ( $\geq 1$ measure of teacher subject knowledge) and student attainment ( $\geq 1$ measure of student achievement).	41 studies (27 suitable for meta-analysis, total of 64 reported effect sizes)	Teachers' subject knowledge is positively associated with student attainment, however the overall magnitude is small. Moderating factors, as well as methodological variation, mean that the relationship is stronger under some conditions/studies than others. Tests of teacher knowledge are more highly correlated with student attainment than are background education factors (such as 'degree level in subject'). The relationship between teachers' subject knowledge and student attainment is stronger in secondary school settings than primary school. The variations in the relationship between teachers' subject matter knowledge and student attainment are not fully explained by the moderating factors (type of measure, level of schooling) and other moderator variables may be involved.	overall $r = .06$ . teacher tests $r = .11$ . primary school $r = .05$ secondary school $r = .07$	although published in the last 10 years this analysis uses some dated studies which may no longer be relevant.
Baker, Gersten & Lee	2002	A synthesis of empirical research on teaching mathematics to low-achieving students	Experimental or quasi-experimental studies (1971-1999) of maths instruction interventions for 'at risk' and/or low achieving students. The intervention must last >90 minutes and $\geq 1$ measure of student performance (+ pre-test) but be analysed.	15 studies (10 randomised, 10 primary school, 5 secondary school)	Positive impact from interventions was found. of the four intervention types: i) providing performance data/feedback to teachers/students; ii) peer tutoring/peer learning; iii) explicit teaching; iv) feedback to parents, the strongest impact was from the use of data and feedback; with moderate to strong impact when performance data was given to teachers, students and parents. Peer assisted learning also showed a substantial moderate impact as did direct explicit teacher instruction, however 'teacher-facilitator' approaches were less successful.	performance data to teachers/ students $d = +0.51$ to $d = +0.71$ . peer assisted learning $d = +0.62$ . explicit teaching $d = +0.65$ . feedback to parents $d = +0.43$	although published in the last 10 years this analysis uses some dated studies which may no longer be relevant.
Ellington	2006	The effects of non-CAS graphing calculators on student achievement and attitude levels in mathematics: A meta-analysis.	Case-control studies comparing maths achievement and attitude in students with access to graphing calculator and students without access to graphing calculators in middle, secondary and higher education.	42 studies (0 randomised, 26 allowed calculators in teaching + assessment, 16 did not allow calculators in assessment)	Use of graphical calculators produced small to medium positive effects upon student achievement and very small positive effects upon attitude (anxiety, self-concept) toward mathematics. Those studies which allowed calculators to be used during assessment as well as teaching produced substantially larger effects than those that didn't, with particular benefit to procedural skills. Conceptual skills were improved with use of graphical calculators in either teaching or teaching/ assessment.	achievement -teaching + assessment $g = +0.29$ . achievement -teaching only $g = +0.19$ . attitude - overall effect $g = +0.06$ to $0.21$ .	
Etsey & Snetzler	1998	A meta-analysis of gender differences in student attitudes toward mathematics.	Studies, of various types, examining maths in the USA between 1970 and 1995 which explored gender differences with use of validated instruments and reported sufficient data for effect sizes to be calculated.	96 studies	Gender differences in attitude to maths do exist but are small in size and may not account for gender differences in maths performance. Studies mainly show more favourable attitudes in males.	attitude $d = -0.01$ , anxiety $d = -0.18$ , confidence $d = +0.18$ . maths viewed as male domain $d = +0.12$ , maths usefulness $d = -0.005$	although published in the last 10 years this paper uses dated data which may no longer be relevant.

Authors	Year	Title	Types Of Studies Reviewed	No. Of Supporting Studies	Findings	Relevant Combined Effect Sizes	Comments
Haas	2005	Teaching methods for secondary algebra: A meta-analysis of findings.	Experimental and quasi-experimental studies (1980-2002) examining the effectiveness of 6 algebra teaching methods related to knowledge and skills development; i) cooperative learning, ii) communication/study skills, iii) technology-aided, iv) problem-based learning, v) manipulatives/models/multiple representations, vi) direct instruction.	35 studies	Direct instruction had the largest effect for low and high ability students. Problem based learning had a large positive effect on pre-algebra classes but a small negative effect upon algebra classes (especially with high ability student). Cooperative learning and manipulative/representations both showed medium sized positive effects, while communication skills and technology assisted approaches showed only small effects.	direct instruction M=+0.55, PBL M=+0.52, Manipulatives M=+0.38, Cooperative Learning M=+0.34, Communication M=+0.07, Technology M=+0.07	
Hurley	2001	Reviewing integrated science and mathematics: the search for evidence and definitions from new perspectives.	Studies, of various types, including mixed method (1935-1997)examining integration of science and mathematics curricula and instruction in interventions at all levels of education.	31 studies (29 with maths achievement data)	The historical scope of this study makes interpretation of findings difficult. Generally integration programmes showed larger effects upon science achievement than maths. Parallel integration, as opposed to sequenced, partial or enhanced integration was less effective. However studies in the 1980s and 1990s show only very small positive effects upon maths.	overall maths d=+0.27, maths effect 1980s/90s d=+0.07	this paper also considers qualitative evidence and historical shifts in research but these are not reported here.
Kroesbergen & Van Luit	2003	Mathematics interventions for children with Special Educational Needs – a meta-analysis.	Experimental or quasi-experimental studies (1985-2000) of the effectiveness of maths instruction interventions for students with mathematical difficulties (but not severe mental disability) in Kindergarten and primary school.	58 studies (21 randomised, 21 use repeated measures design)	Interventions aimed at improving basic skills were seen to be most effective, with moderate to large effects. Duration of the intervention was important, but so too was instructional method. Direct instruction and self-instruction were most effective, mediated instruction was less so. Computer assisted instruction and peer tutoring showed substantial positive effects, but some interventions not including these showed larger effects.	direct instruction d=+0.91, self- instruction d=+0.1.45, mediated/assisted d=+0.34, peer tutoring d=+0.87, computer assisted d=+0.51	
Kunsch, Jitendra & Sood	2007	The effects of peer-mediated instruction in mathematics for students with learning problems: A research Synthesis.	Experimental or quasi-experimental studies (1978-2006) evaluating peer-mediated interventions in mathematics for students 'at risk' or with disabilities and ≥1 measure of student maths achievement.	17 studies (majority, 82%, focus on primary school)	Substantial positive effects from peer mediated interventions for at risk/disabled students are evident, with stronger effects for primary school interventions than for secondary school. Stronger effects were seen for at risk students than for students with disabilities and for regular classrooms rather than special education classrooms.	overall d=+0.47, primary school d=+0.57, secondary school d=+0.18, at risk students d=+0.66, students with disability d=+0.21, general classroom d=+0.56, special education classroom d=+0.32	

Authors	Year	Title	Types Of Studies Reviewed	No. Of Supporting Studies	Findings	Relevant Combined Effect Sizes	Comments
Ma	1999	A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics.	Studies, of various types (1975-1999) examining the relationship between anxiety toward maths and maths achievement in primary and secondary students.	26 studies	There is a significant, moderate relationship between , maths anxiety and poor maths performance. This is consistent across gender, grade, ethnicity and different measures of anxiety. However, perhaps surprisingly, the relationship is weaker for standardised assessment of maths than for teachers' grades and non-standardised assessment.	overall $r = -0.27$	
Springer, Stanne & Donovan	1997	Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A Meta-analysis.	Studies, of various types, (1980-1997), of small group (2 to 10 students) cooperative or collaborative learning in undergraduate education, including maths students. The studies examine achievement, persistence and attitudes.	57 studies (22 focus on maths, all examine achievement and 5 examine attitude)	Various forms of small group learning are effective across subject areas, maths effect sizes are similar to others. Moderate effect sizes are seen for impact on maths achievement and attitude toward learning (including self-esteem) in university.	maths achievement $d = +0.53$ , attitude in maths students $d = +0.43$	
Wang, Jiao, Young, Brooks & Olson	2007	A meta-analysis of testing mode effects in Grade K-12 mathematics tests.	Experimental or quasi-experimental studies (1980-2005) comparing computer Based Tests (CBT) with pen and paper tests (PPT) in mathematic s achievement.	44 studies (29 randomised, 25 use repeated measures design)	Overall PPT produced slightly better results, however the diversity of results in studies was very large, when statistical adjustments were made no significant difference between PPT and CBT was seen. A small effect was seen in that larger differences did exist for linear PPT and CBT tests than when both PPT and CBT were adaptive tests.	overall $d = -0.06$ (n.s.)	

## Systematic Reviews by Sector

Authors	Year	Country of publication	Title	Scope	Papers reviewed	Findings	No. & Strength of supporting studies
Kyriacou, Golding & the Mathematics Education Review Group	2006	United Kingdom	Mathematics education: A systematic review of strategies to raise pupils' motivational effort in Key Stage 4 mathematics.	Systematic Review: Comprehensive review of effectiveness of interventions/strategies to raise pupil motivation in maths in years 7 to 10. Four areas identified: i) grouping ii) pupil identity iii) teaching for engagement, and iv) innovative methods.	25 studies of mixed type and quality. Of this 16 were 'evaluation' studies (none randomised, 13 naturally occurring evaluations) and 8 were correlational studies. Overall only one study was rated as high quality, 8 considered medium quality and 16 were rated low quality.	<ol style="list-style-type: none"> <li>Grouping by ability and gender were not consistently associated with any effect upon motivation. Student awareness of ability grouping was clear, with lower sets aware that their grades were limited and thus it was difficult to sustain motivation.</li> <li>Raising student motivational effort through developing more positive student mathematical identity uses strategies characterised by i) caring and supportive classroom environment, ii) activities which students find enjoyable and challenging, iii) focus on the development of deep mathematical understanding, iv) opportunities for student to collaborate, and v) making students feel equally valued.</li> <li>Teaching for engagement is characterised by similar themes to those listed for developing student identity, however, greater emphasis is on teacher care and support and making maths enjoyable.</li> <li>Innovative methods focusing on ICT use have a powerful impact upon student motivation. Two stages of impact are identified i) a novelty effect, and ii) an effect through development of deeper learning. Longitudinal impact requires a focus on the second. There is also some evidence that cognitive acceleration programmes (e.g. CAME), self-regulation strategies and innovative formative assessment can play a part in raising motivation.</li> </ol>	5 studies, unknown quality  3 studies, unknown quality
Slavin & Lake	2007	USA	Effective programmes in elementary mathematics: a best evidence synthesis.	Best Evidence Synthesis of : Examined elementary (K-5) maths outcomes in relation to all types of maths programmes available today: i) maths curricula (textbook based) ii) supplementary CAI iii) instructional process or classroom practice programmes.	87 experimental studies (randomised (36) or matched-control (51)) looking at outcomes measures not inherent to the experimental treatment over >12 weeks.	<ol style="list-style-type: none"> <li>Limited evidence of differential effects of curricula in text books - median effect size (ES=+.10). Strongest effect found with Everyday mathematics (ES=+.34) in schools using it for &gt;4yrs.</li> <li>Moderate effects of CAI - most studies find positive effect, especially on maths computation. Median ES=+.19, strong considering relatively rigorous studies and limited intervention time (usually 30min/week). CAI seen as good supplementary strategy.</li> <li>Strong positive effect from instructional processes with median ES=+.33. Of 7 processes studied; cooperative learning (e.g. Classroom Peer Tutoring, ES=+.33) &amp; professional development programmes in classroom management and motivation programmes (e.g. Missouri Mathematics Programme, ES=+.29) were most effective.</li> </ol>	13 studies, only 2 randomised. Considered marginal methodological quality  38 studies, 15 randomised. Considered moderate quality  36 studies, 19 randomised. Considered high quality



Authors	Year	Country of publication	Title	Scope	Papers reviewed	Findings	No. & Strength of supporting studies
Slavin, Lake & Groff	2007	USA	Effective programmes in middle and high school mathematics: a best evidence synthesis.	Best Evidence Synthesis of : Examined middle and high school maths outcomes in relation to all types of maths programmes available today: i) maths curricula (textbook based) ii) supplementary CAI (iii) instructional process or classroom practice programmes.	97 experimental studies (randomised (25) or matched-control (72)) looking at outcomes measures not inherent to the experimental treatment over >12 weeks.	<p>1. As with elementary maths review, there was limited evidence of differential effects of text books -small median effect size (ES=+ 05). Strongest effect found with Everyday mathematics (ES= + .34) in schools using it for &gt;4yrs.</p> <p>2. Moderate effects of CAI - most studies find +ve effect, especially on maths computation. Median ES= +.18, strong considering relatively rigorous studies and limited intervention time (usually 30min/week). No programme stood out as having more notable effects and effect for 'core', 'supplementary' and 'computer-managed learning' approaches were equivalent. These results were very similar to those from the elementary maths review and CAI is seen as a good strategy.</p> <p>3. Strong positive effect from instructional processes varied with cooperative learning (median ES= +.32 in 8 studies) proving the most effective. Programmes such as STAD and IMPROVE showed very strong effects in randomised trials (ES= +.50).</p>	<p>38 studies, 8 randomised. Considered mixed quality.</p> <p>37 studies, 8 randomised. Considered moderate to strong quality.</p> <p>22 studies, 5 randomised. Considered moderate to strong quality.</p>
The EPPI mathematics Education Review Group	2004	United Kingdom	A systematic review of the impact of the daily mathematics lesson in enhancing pupil confidence and competence in early mathematics.	Systematic Review: Specific review of research relating to introduction of the UK National Numeracy Strategy's daily maths lesson in K-3 ( a 45-60 minute, 3 part lesson with emphasis on whole-class teaching).	18 high quality studies identified for in-depth analysis from an initial trawl which produced 743.	<p>1. Key features of the daily maths lesson were well received by teachers and widely implemented. However, the intention that whole-class teaching be interactive, was not realised and there is also some evidence that some implemented approaches were more 'traditional' and undermined development of a more strategic and reflective approach by teachers and may have disadvantaged lower attaining pupils.</p> <p>2. Some evidence of enhanced pupil confidence and competence in early maths. However, there is also some evidence that the stricter time management posed particular problems for lower attaining students and the overall gains in student competence may have been due to improved alignment between the lesson and maths assessment, rather than broader gains in student understanding.</p>	<p>18 studies, classified as evaluation or description studies. Considered marginal quality. Tended to explore relationships rather than examine causality.</p>

Authors	Year	Country of publication	Title	Scope	Papers reviewed	Findings	No. & Strength of supporting studies
Torgerson, Porthouse, & Brooks	2005	United Kingdom	A Systematic review of controlled trials evaluating interventions in adult literacy and numeracy.	Systematic Review: Examined adult education in literacy and numeracy published between 1980 and 2002. Papers included in the review had to be controlled trials evaluating interventions aimed at increasing the literacy and numeracy in study populations of adults.	A total of 4,555 potential relevant papers were identified and after various exclusions 59 papers were coded. Of these 27 controlled trials which evaluated strategies and pedagogies designed to increase adult literacy and numeracy were included. Of these nine control trials were examined for this paper, 3 examined numeracy outcomes.	1. Inconclusive: In this systematic review nine relevant control trials of interventions for adult literacy and numeracy where reviewers were able to calculate the effect sizes from the data included in the studies. There have been few attempts to expose common adult literacy or numeracy programmes to rigorous evaluation. The review does provide a strong steer for the direction to be taken by educational researchers. Because of the present inadequate evidence base rigorously designed randomised controlled trials and quasi-experiments are required.	18 controlled trials without effect sizes, 9 control trials complete data, no randomised studies
Anthony & Walshaw	2007	New Zealand	Effective pedagogy in Mathematics/ Pangarau.	Best Evidence Synthesis: Aims to 1. identify and explain the characteristics of pedagogical approaches that enhance Mathematics/ Pangarau proficiency in NZ. 2. identify pedagogical approaches that make a significant difference for diverse learners in early childhood and school years in Mathematics/ Pangarau in NZ.	total number of sources cited was over 1,100. A total of 660 papers are referred to in the report. Including non-empirical discussion documents. Evidence spanned sectors: preschools (18%), primary (48%), secondary (21%), teacher education (6%).	1. Sound teacher subject matter knowledge and pedagogical content knowledge are required to assess student understanding and for adapting tasks, activities and resources to build students' mathematical knowledge. Effective teachers use their subject matter knowledge to make decisions on lesson content that provide opportunities for students to develop their mathematical understandings and their cultural identity. 2. Building home-community and school-centre partnerships has a positive effect. 3. Early childhood research proposes that a balance between teacher initiated numeracy group work and student initiated instructive numeracy play activities is beneficial. Within childcare centres and schools effective teachers create opportunities for students to develop their mathematical and cultural identities. 4. There is conclusive evidence that effective mathematics teaching bridges the gap between learners' intuitive understanding and the conventional mathematics understandings sanctioned by the world at large.	Research quality not commented on. Evidence drawn from the full range of research methodologies. No effect sizes given.

## ICT & CAI meta-analyses

Authors	Year	Title	Scope	Papers Reviewed	Findings	No. & Strength Of Supporting Studies
Kyriacou, Golding & the Mathematics Education Review Group	2006	Mathematics education: A systematic review of strategies to raise pupils' motivational effort in Key Stage 4 mathematics.	Systematic Review: Comprehensive review of effectiveness of interventions/strategies to raise pupil motivation in maths in years 7 to 10. Four areas identified: i) grouping ii) pupil identity iii) teaching for engagement, and iv) innovative methods	25 studies of mixed type and quality. Of this 16 were 'evaluation' studies (none randomised, 13 naturally occurring evaluations) and 8 were correlational studies. Overall only one study was rated as high quality, 8 considered medium quality and 16 were rated low quality.	Innovative methods focusing on ICT use have a powerful impact upon student motivation. Two stages of impact are identified i) a novelty effect, and ii) an effect through development of deeper learning. Longitudinal impact requires a focus on the second, there is also some evidence that cognitive acceleration programmes (e.g. CAME), self-regulation strategies and innovative formative assessment can play a part in raising motivation.	36 studies, 19 randomised. Considered high quality
Slavin & Lake	2007	Effective programmes in elementary mathematics: a best evidence synthesis.	Best Evidence Synthesis of : Examined elementary (K-5) maths outcomes in relation to all types of maths programmes available today: i) maths curricula (textbook based) ii) supplementary CAI iii) instructional process or classroom practice programmes.	87 experimental studies (randomised (36) or matched-control (51)) looking at outcomes measures not inherent to the experimental treatment over >12 weeks.	Moderate effects of CAI - most studies find positive effect, especially on maths computation. Median ES=+.19, strong considering relatively rigorous studies and limited intervention time (usually 30min/week). CAI seen as good supplementary strategy.	38 studies, 8 randomised. Considered mixed quality. 37 studies, 8 randomised. Considered moderate to strong quality.
Slavin, Lake & Groff	2007	Effective programmes in middle and high school mathematics: a best evidence synthesis.	Best Evidence Synthesis of : Examined middle and high school maths outcomes in relation to all types of maths programmes available today: i) maths curricula (textbook based) ii) supplementary CAI iii) instructional process or classroom practice programmes.	97 experimental studies (randomised (25) or matched-control (72)) looking at outcomes measures not inherent to the experimental treatment over >12 weeks.	Moderate effects of CAI - most studies find +ve effect, especially on maths computation. Median ES=+.18, strong considering relatively rigorous studies and limited intervention time (usually 30min/week). No programme stood out as having more notable effects and effect for 'core', 'supplementary' and 'computer-managed learning' approaches were equivalent. These results were very similar to those from the elementary maths review and CAI is seen as a good strategy.	22 studies, 5 randomised. Considered moderate to strong quality. 18 studies, classified as evaluation or description studies. Considered marginal quality. Tended to explore relationships rather than examine causality.

Systematic Reviews & Best Evidence Syntheses

Meta-analyses

Authors	Year	Title	Scope	Papers Reviewed	Findings	No. & Strength Of Supporting Studies
Ellington	2006	The effects of non-CAS graphing calculators on student achievement and attitude levels in mathematics: A meta-analysis.	Case-control studies comparing maths achievement and attitude in students with access to graphing calculator and students without access to graphing calculators middle, secondary and higher education.	42 studies (0 randomised, 26 allowed calculators in teaching + assessment, 16 did not allow calculators in assessment)	Use of graphical calculators produced small to medium positive effects upon student achievement and very small positive effects upon attitude (anxiety, self-concept) toward mathematics. Those studies which allowed calculators to be used during assessment as well as teaching produced substantially larger effects than those that didn't, with particular benefit to procedural skills. Conceptual skills were improved with use of graphical calculators in either teaching or teaching/ assessment.	achievement -teaching + assessment g= +0.29, achievement -teaching only g=+0.19, attitude - overall effect g=+0.06 to 0.21.
Haas	2005	Teaching methods for secondary algebra: A meta-analysis of findings.	Experimental and quasi-experimental studies (1980-2002) examining the effectiveness of 6 algebra teaching methods related to knowledge and skills development; i) cooperative learning, ii) communication/study skills, iii) technology-aided, iv) problem-based learning, v) manipulatives/models/multiple representations, vi) direct instruction.	35 studies	Direct instruction had the largest effect for low and high ability students. Problem based learning had a large positive effect on pre-algebra classes but a small negative effect upon algebra classes (especially with high ability student). Cooperative learning and manipulative/representations both showed medium sized positive effects, while communication skills and technology assisted approaches showed only small effects.	direct instruction M= +0.55, PBL M=+0.52, Manipulatives M=+0.38, Cooperative Learning M=+0.34, Communication M=+0.07, Technology M=+0.07
Kroesbergen & Van Luit	2003	Mathematics interventions for children with Special Educational Needs – a meta-analysis.	Experimental or quasi-experimental studies (1985-2000) of the effectiveness of maths instruction interventions for students with mathematical difficulties (but not severe mental disability) in Kindergarten and primary school.	58 studies (21 randomised, 21 use repeated measures design)	Interventions aimed at improving basic skills were seen to be most effective, with moderate to large effects. Duration of the intervention was important, but so too was instructional method. Direct instruction and self-instruction were most effective, mediated instruction was less so. Computer assisted instruction and peer tutoring showed substantial positive effects, but some interventions not including these showed larger effects.	direct instruction d=+0.91, self-instructed/assisted d=+0.34, peer tutoring d=+0.87, computer assisted d=+0.51
Wang, Jiao, Young, Brooks & Olson	2007	A meta-analysis of testing mode effects in Grade K-12 mathematics tests.	Experimental or quasi-experimental studies (1980-2005) comparing computer Based tests (CBT) with pen and paper tests (PPT) in mathematics achievement.	44 studies (29 randomised, 25 use repeated measures design)	Overall PPT produced slightly better results, however the diversity of results in studies was very large, when statistical adjustments were made no significant difference between PPT and CBT was seen. A small effect was seen in that larger differences did exist for linear PPT and CBT tests than when both PPT and CBT were adaptive tests.	overall d=-0.06 (n.s.)

## At risk & special needs meta-analyses

Authors	Year	Title	Types Of Studies Reviewed	No. Of Supporting Studies	Findings	Relevant Combined Effect Sizes	Comments
Baker, Gersten & Lee	2002	A synthesis of empirical research on teaching mathematics to low-achieving students	Experimental or quasi-experimental studies (1971-1999) of maths instruction interventions for 'at risk' and/or low achieving students. The intervention must last >90 minutes and ≥1 measure of student performance (+ pre-test) but be analysed.	15 studies (10 randomised, 10 primary school, 5 secondary school)	Positive impact from interventions was found. of the four intervention types: i) providing performance data/feedback to teachers/students; ii) peer tutoring/peer learning, iii) explicit teaching, i v) feedback to parents; the strongest impact was from the use of data and feedback; with moderate to strong impact when performance data was given to teachers, students and parents. Peer assisted learning also showed a substantial moderate impact as did direct explicit teacher instruction, however 'teacher-facilitator' approaches were less successful.	performance data to teachers/ students $d=+0.51$ to $d=+0.71$ . peer assisted learning $d=+0.62$ . explicit teaching $d=+0.65$ . feedback to parents $d=+0.43$	although published in the last 10 years this analysis uses some dated studies which may no longer be relevant
Kroesbergen & Van Luit	2003	Mathematics interventions for children with Special Educational Needs – a meta-analysis.	Experimental or quasi-experimental studies (1985-2000) of the effectiveness of maths instruction interventions for students with mathematical difficulties (but not severe mental disability) in Kindergarten and primary school.	58 studies (21 randomised, 21 use repeated measures design)	Interventions aimed at improving basic skills were seen to be most effective, with moderate to large effects. Duration of the intervention was important, but so too was instructional method. Direct instruction and self-instruction were most effective, mediated instruction was less so. Computer assisted instruction and peer tutoring showed substantial positive effects, but some interventions not including these showed larger effects.	direct instruction $d=+0.91$ , self-instruction $d=+0.145$ , mediated/assisted $d=+0.34$ , peer tutoring $d=+0.87$ , computer assisted $d=+0.51$	
Kunsch, Jitendra & Sood	2007	The effects of peer-mediated instruction in mathematics for students with learning problems: A research synthesis.	Experimental or quasi-experimental studies (1978-2006) evaluating peer-mediated interventions in mathematics for students 'at risk' or with disabilities and ≥1 measure of student maths achievement.	17 studies (majority 82%, focus on primary school)	Substantial positive effects from peer mediated interventions for at risk/disabled students are evident, with stronger effects for primary school interventions than for secondary school. Stronger effects were seen for at risk students than for students with disabilities and for regular classrooms rather than special education classrooms.	overall $d=+0.47$ , primary school $d=+0.57$ , secondary school $d=+0.18$ , at risk students $d=+0.66$ , students with disability $d=+0.21$ , general classroom $d=+0.56$ , special education classroom $d=+0.32$	



## Appendix 3: List of submissions

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### Organisation

- 1 Curriculum Corporation
- 2 The INISSS Project - Department of Education Tasmania
- 3 Flinders University - School of Education
- 4 Catholic Education South Australia
- 5 Catholic Education - Archdiocese of Brisbane
- 6 Australian Technology Network of Universities
- 7 Australian Primary Principals Association
- 8 Professor Jane Watson, University of Tasmania
- 9 Australian Education Union
- 10 Ministry of Education - New Zealand
- 11 Australian Council of Deans of Science
- 12 Queensland College of Teachers
- 13 Dr Kim Beswick, University of Tasmania
- 14 Dr Ann Gervasoni, Australian Catholic University
- 15 Dr Rosemary Callingham, University of New England
- 16 National Independent Special Schools Association
- 17 Catholic Education Office - Diocese of Darwin
- 18 The Association of Independent Schools of Victoria
- 19 Australian Association of Special Education – New South Wales Chapter
- 20 Australian Council for Educational Research
- 21 Department of Education & Children's Services South Australia
- 22 Catholic Education Commission New South Wales
- 23 Professor Dianne Seimon, Royal Melbourne Institute of Technology University
- 24 Australian Academy of Technological Sciences & Engineering
- 25 The National Centre of Science, ICT, and Mathematics Education for Rural and Regional Australia (SiMERR)
- 26 Association of Independent Schools of South Australia
- 27 Australian Capital Territory Department of Education & Training
- 28 Department of Education Science and Training
- 29 Australian Parents Council
- 30 Queensland Catholic Education Commission
- 31 Australian Association of Mathematics Teachers
- 32 Curriculum Council of Western Australia
- 33 Australian Joint Council of Professional Teaching Associations
- 34 New South Wales Board of Studies
- 35 New South Wales Department of Education & Training
- 36 Teachers Registration Board, Tasmania
- 37 Northern Territory Department of Education, Employment & Training
- 38 Mathematics Education Research Group of Australasia
- 39 Education Queensland

## Organisation

- 40** Department of Education & Training Western Australia
- 41** MCEETYA Reference Group on Indigenous Education – WA
- 42** Mr Dave Tout, Centre for Adult Education
- 43** Tasmanian Department of Education
- 44** Archdiocese of Canberra and Goulburn Catholic Education Office
- 45** Independent Schools Queensland
- 46** National Centre for Vocational Education Research Limited
- 47** Dr Kenneth Rowe, Australian Council for Educational Research
- 48** Victorian Department of Education and Victorian Curriculum and Assessment Authority
- 49** Australian Mathematical Sciences Institute
- 50** Catholic Education Commission of Victoria
- 51** The Association of Independent Schools of New South Wales Limited
- 52** Australian Centre for Educational Studies, Macquarie University
- 53** Independent Education Union of Australia



